Peakedness and peakedness ordering in symmetric distributions

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Abstract: There are many ways to measure the dispersion of a random variable. One such method uses the concept of peakedness. If the random variable X is symmetric about a point μ, then Birnbaum (1948) defined the function $\mathcal{P}_\mu(x) = P(|X-\mu| \leq x),\ x \geq 0$, as the peakedness of X. If two random variables, X and Y, are symmetric about the points μ and ν, respectively, then X is said to be less peaked than Y, denoted by $X \leq_{pkd(\mu,\nu)} Y$, if $P(|X-\mu| \leq x) \leq P(|Y-\nu| \leq x)$ for all $x \geq 0$, i.e., $|X-\mu|$ is stochastically larger than $|Y-\nu|$. For normal distributions this is equivalent to variance ordering. Peakedness ordering can be generalized to the case where μ and ν are arbitrary points. However, in this paper we study the comparison of dispersions in two symmetric and continuous random variables using the peakedness concept where normality, and even moment assumptions are not necessary. We provide estimators of the distribution functions under the restriction of symmetry and peakedness ordering, show that they are consistent, derive the weak convergence of the estimators, compare them with the empirical estimators, and provide formulas for statistical inferences. An example is given to illustrate the theoretical results.