19.1) a) There are 2 levels of factor A and 4 levels of B, so there are 8 treatments in all.
b) Based on the description, the response variable is “Infection risk”.

19.2) In a multi-factor study, we need to consider the treatments as representing various combinations of levels of the factors, so that we can separate out the effect of each individual factor on the response and so we can assess whether there is interaction among the factors.

19.3) \((\alpha\beta)_{11} = 9 - 13 - 9 + 12 = -1.\) \((\alpha\beta)_{12} = 12 - 13 - 11 + 12 = 0.\) \((\alpha\beta)_{13} = 18 - 13 - 16 + 12 = 1.\) \((\alpha\beta)_{21} = 9 - 11 - 9 + 12 = 1.\) \((\alpha\beta)_{22} = 10 - 11 - 11 + 12 = 0.\) \((\alpha\beta)_{23} = 14 - 11 - 16 + 12 = -1.\)

19.4) a) \(\mu_1 = (34+23+36)/3 = 31.\) \(\mu_2 = (40+29+42)/3 = 37.\)
b) The overall mean \(\mu = (34+23+36+40+29+42)/6 = 34.\)
So \(\alpha_1 = 31 - 34 = -3\) and \(\alpha_2 = 37 - 34 = 3.\)
c) No. Those quantities measure the effect on the mean response of moving across the various levels of B, while A is fixed at level 1. An interaction effect measures the effect on the mean response of changing the levels of one factor, at differing levels of the other factor.
d) We see from the interaction plot below that A and B do not interact.

19.5) a) \(\mu_1 = (250+288)/2 = 269.\) \(\mu_2 = (265+273)/2 = 269.\) \(\mu_3 = (268+270)/2 = 269.\) \(\mu_4 = (269+269)/2 = 269.\) And the overall mean \(\mu = 269.\) So \(\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0.\) This implies that the mean response is the same at each level of factor B (all of factor B’s main effects are 0).
b) We see from the interaction plot that the difference in mean response for the two levels of A are quite different depending on which level that factor B is at. It appears the interaction is important: When B is at level 1, there is a wide difference between the mean responses at levels 1 and 2 of A, but when B is at levels 3 or 4, there is hardly any difference between the mean responses at levels 1 and 2 of A.
19.7) a) \(E\text{(MSE)} = 1.96, \ E\text{(MSA)} = 1.96 + (3)10[(31 - 34)^2 + (37 - 34)^2] / 1 = 541.96\).

b) Yes, \(E\text{(MSA)}\) is substantially larger than \(E\text{(MSE)}\). This implies that we are highly likely to obtain a significant \(F^*\) and conclude the main effects of \(A\) are not all zero.

19.13) b) From SAS:

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type I SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>eyecontact</td>
<td>1</td>
<td>54.45000000</td>
<td>54.45000000</td>
<td>8.96</td>
<td>0.0086</td>
</tr>
<tr>
<td>gender</td>
<td>1</td>
<td>76.05000000</td>
<td>76.05000000</td>
<td>12.52</td>
<td>0.0027</td>
</tr>
<tr>
<td>eyecontact*gender</td>
<td>1</td>
<td>1.25000000</td>
<td>1.25000000</td>
<td>0.21</td>
<td>0.6562</td>
</tr>
<tr>
<td>Error</td>
<td>16</td>
<td>97.20000000</td>
<td>6.07500000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corrected Total</td>
<td>19</td>
<td>228.9500000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Based on the sums of squares, no particular source accounts for a huge portion of the total variability.

c) We test \(H_0: (\alpha \beta)_i = 0\) for all \(i, j\) vs. \(H_a: (\alpha \beta)_i \neq 0\) for some \(i, j\). Since \(F^* = 0.21 < F_{(99, 1, 16)} = 8.6\), we fail to reject \(H_0\). We conclude there is not significant interaction between eye contact and gender. The \(P\)-value is 0.6562.

d) Testing for eye contact main effects, we test \(H_0: \alpha_i = \alpha_j = 0\) vs. \(H_a: \alpha_i \neq 0\) for some \(i\). Since \(F^* = 8.96 > F_{(99, 1, 16)} = 8.6\), we reject \(H_0\). We conclude there is a significant effect of eye contact on rating. The \(P\)-value is 0.0086.

Testing for gender main effects, we test \(H_0: \beta_1 = \beta_2 = 0\) vs. \(H_a: \beta_1 \neq 0\) for some \(j\). Since \(F^* = 12.52 > F_{(99, 1, 16)} = 8.6\), we reject \(H_0\). We conclude there is a significant effect of gender on rating. The \(P\)-value is 0.0027.

It is meaningful to test for main effects in this case since we have no significant interaction.

19.16) c) Based on the residual plot (shown below), the data seem to have relatively similar variance across the treatments. The equal-variances assumption seems reasonable overall. Also: Brown-Forsythe test shows a \(P\)-value of 0.9836. So we can conclude that the equal-variances assumption is reasonable.
d) Based on the normal Q-Q plot for the residuals (shown below), the normality assumption seems somewhat reasonable. There is a slight, but not major, curvature in the Q-Q plot. Shapiro-Wilk test gives a P-value of 0.2712, so the normality assumption is definitely reasonable.

19.17)a)
There is clear interaction here, leading us to suspect that the interaction effects are significant. This makes it difficult/impossible to say anything about the effect of each factor in isolation.
b) From SAS:

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Type I SS</th>
<th>Mean Square</th>
<th>F Value</th>
<th>Pr &gt; F</th>
</tr>
</thead>
<tbody>
<tr>
<td>tech</td>
<td>2</td>
<td>24.577778</td>
<td>12.288889</td>
<td>0.24</td>
<td>0.7908</td>
</tr>
</tbody>
</table>
Based on the sums of squares, the interaction seems to account for a large portion of the variability.

c) We test $H_0$: $(\alpha \beta)_{ij} = 0$ for all $i, j$ vs. $H_a$: $(\alpha \beta)_{ij} \neq 0$ for some $i, j$. Since $F^* = 5.84 > F_{(99,4,30)} \approx 3.9$, we reject $H_0$. We conclude there is significant interaction between technician and make. The $P$-value is 0.001.

d) Testing for technician main effects, we test $H_0$: $\alpha_1 = \alpha_2 = \alpha_3 = 0$ vs. $H_a$: $\alpha_i \neq 0$ for some $i$. Since $F^* = 0.24 < F_{(99,2,30)} \approx 5.3$, we fail to reject $H_0$. We conclude there is no significant effect of technician on service time. The $P$-value is 0.7908.

Testing for make main effects, we test $H_0$: $\beta_1 = \beta_2 = \beta_3 = 0$ vs. $H_a$: $\beta_j \neq 0$ for some $j$. Since $F^* = 0.27 < F_{(99,2,30)} \approx 5.3$, we fail to reject $H_0$. We conclude there is no significant effect of make on service time. The $P$-value is 0.7633.

It is not meaningful to test for main effects in this case since we have significant interaction.

19.31) b) The 99% CI for $\mu_1$ is (9.12, 13.68). This implies that the population mean rating for applicants when eye contact is present is between 9.12 and 13.68, with 99% confidence.

19.33) a) From SAS, a 99% CI for $\mu_{ij}$ is (51.03, 68.57). With 99% confidence, the mean service time for drives with technician 1 and make 1 is between 51.03 and 68.57 minutes.

b) The 99% CI for $\mu_{22} - \mu_{21}$ is (0.396, 25.204). This implies that, with 99% confidence, the mean service time for drives with technician 2 and make 2 is between 0.4 minutes more and 25.2 minutes more than the mean service time for drives with technician 2 and make 1.

c) See last pages for solution.

e) See last pages for solution.

19.48) See last page for solution.

20.1) Since SSE has ab(n - 1) degrees of freedom, when n = 1 there are zero degrees of freedom associated with SSE. This is a problem because we cannot calculate MSE for this model since we’d be dividing by zero.

20.2) a) It does not appear that A and B interact notably. There do appear to be significant main effects for both A and B. The mean responses vary a good bit across the levels of A, and there is a sizable shift in mean response across the two levels of B.

b) Testing for location main effects, we test $H_0$: $\alpha_1 = \alpha_2 = \alpha_3 = 0$ vs. $H_a$: $\alpha_i \neq 0$ for some $i$. Since $F^* = 107.26 > F_{(99,3,3)} = 9.28$, we reject $H_0$. We conclude there is a significant effect of location on hours. The $P$-value is 0.0015.
Testing for week main effects, we test $H_0$: $\beta_1 = \beta_2 = 0$ vs. $H_a$: $\beta_j \neq 0$ for some $j$. Since $F^* = 409.09 > F_{(95, 1, 3)} = 10.1$, we reject $H_0$. We conclude there is a significant effect of week on hours. The P-value is 0.0003.

20.4) We test $H_0$: "no interaction is present" vs. $H_a$: "interaction is present". The P-value SAS gives for Tukey's test of additivity is 0.523. Therefore we fail to reject $H_0$ and conclude that the additive model is fine; we have no evidence of interaction. If the additive model were not appropriate, I might transform the response variable to see if that would remove the interaction.
19.33 c) Let $L_1 = \mu_{11} - \mu_{12}$
\[= (\mu_{11} + \alpha_1 + \beta_1 + (\alpha \beta)_{11}) - (\mu_{12} + \alpha_1 + \beta_2 + (\alpha \beta)_{12})
= \beta_1 - \beta_2 + (\alpha \beta)_{11} - (\alpha \beta)_{12}\]

Similarly, $L_2 = \mu_{11} - \mu_{13} = \beta_1 - \beta_3 + (\alpha \beta)_{11} - (\alpha \beta)_{13}$
and $L_3 = \mu_{12} - \mu_{13} = \beta_2 - \beta_3 + (\alpha \beta)_{12} - (\alpha \beta)_{13}$

We test $H_0: L_1 = 0, H_0: L_2 = 0, H_0: L_3 = 0$ with the Bonferroni method at family confidence level 0.95.

From SAS, $\hat{L}_1 = 12.0, \hat{L}_2 = 1.4, \hat{L}_3 = -10.6,$
and $s(\hat{L})$ for each is 4.561.

Since $g=3$ tests, the Bonferroni correction yields $t(1 - \frac{0.05}{2(3)}, 36) = t(0.9917, 36) \approx 2.51$

So: $\frac{|\hat{L}_1|}{s(\hat{L}_1)} = 2.63 > 2.51, \frac{|\hat{L}_2|}{s(\hat{L}_2)} = 0.31 < 2.51, \frac{|\hat{L}_3|}{s(\hat{L}_3)} = 2.32 < 2.51.$

So we conclude for technician 1, the mean service times for make 1 and for make 2 significantly differ. But the mean times for make 1 and make 3, and for make 2 and make 3, do not significantly differ.
The time savings (on average) can be expressed as:

\[ L = \frac{\mu_{11} + \mu_{12} + \mu_{13} + \mu_{21} + \mu_{22} + \mu_{23} + \mu_{31} + \mu_{32} + \mu_{33}}{9} - \frac{\mu_{12} + \mu_{21} + \mu_{33}}{3} \]

\[ L = \frac{1}{q} \mu_{11} - \frac{2}{q} \mu_{12} + \frac{1}{q} \mu_{13} - \frac{2}{q} \mu_{21} + \frac{1}{q} \mu_{22} + \frac{1}{q} \mu_{23} + \frac{1}{q} \mu_{31} + \frac{1}{q} \mu_{32} - \frac{2}{q} \mu_{33} \]

which reduces to (in terms of the factor effects):

\[ \frac{1}{q} (\alpha \beta)_{11} - \frac{2}{q} (\alpha \beta)_{12} + \frac{1}{q} (\alpha \beta)_{13} - \frac{2}{q} (\alpha \beta)_{21} + \frac{1}{q} (\alpha \beta)_{22} + \frac{1}{q} (\alpha \beta)_{23} + \frac{1}{q} (\alpha \beta)_{31} + \frac{1}{q} (\alpha \beta)_{32} - \frac{2}{q} (\alpha \beta)_{33} \]

since the \( \alpha_i \) and \( \beta_j \) terms can be seen to cancel out.

From SAS, a 99% CI for this contrast is (3.09, 11.36).
\[ E \left[ \sum_j c_j \bar{Y}_{.j} \right] = \sum_j c_j E[\bar{Y}_{.j}] \]

\[
E[\bar{Y}_{.j}] = E \left[ \frac{\sum_{i=1}^a \sum_{k=1}^n Y_{ijk}}{an} \right] = \frac{1}{an} \sum_{i=1}^a \sum_{k=1}^n E[Y_{ijk}] = \frac{1}{an} \sum_{i=1}^a n \mu_{ij} = \frac{1}{a} \sum_{i=1}^a \mu_{ij} = \mu_{.j}
\]

Hence \[ E \left[ \sum_j c_j \bar{Y}_{.j} \right] = \sum_j c_j \mu_{.j} \] and thus the estimator is unbiased.

\[
\text{var} \left[ \sum_j c_j \bar{Y}_{.j} \right] = \sum_j c_j^2 \text{var} [\bar{Y}_{.j}] \]

\[
\text{var} [\bar{Y}_{.j}] = \text{var} \left[ \frac{\sum_{i=1}^a \sum_{k=1}^n Y_{ijk}}{an} \right] = \frac{1}{a^2 n^2} \sum_{i=1}^a \sum_{k=1}^n \text{var} [Y_{ijk}] = \frac{1}{a^2 n^2} \sum_{i=1}^a \sum_{k=1}^n \sigma^2 = \frac{\sigma^2}{an}
\]

Hence \[ \text{var} \left[ \sum_j c_j \bar{Y}_{.j} \right] = \frac{\sigma^2}{an} \sum_j c_j^2 \]

(since the \( Y \) values are independent)