$$E[h(X, Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x, y)f(x, y)dxdy \tag{2.A.1}$$

As a corollary to Equation (2.A.1), we easily obtain the important result

$$E(aX + bY + c) = aE(X) + bE(Y) + c \tag{2.A.2}$$

We also have

$$E(XY) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xyf(x, y)dxdy \tag{2.A.3}$$

The **variance** of a random variable X is defined as

$$Var(X) = E\{[X - E(X)]^2\} \tag{2.A.4}$$

(provided $E(X^2)$ exists). The variance of X is often denoted by σ^2 or σ_X^2.

Properties of Variance

$$Var(X) \geq 0 \tag{2.A.5}$$

$$Var(a + bX) = b^2 Var(X) \tag{2.A.6}$$

If X and Y are independent, then

$$Var(X + Y) = Var(X) + Var(Y) \tag{2.A.7}$$

In general, it may be shown that

$$Var(X) = E(X^2) - [E(X)]^2 \tag{2.A.8}$$

The positive square root of the variance of X is called the **standard deviation** of X and is often denoted by σ or σ_X. The random variable $(X - \mu_X)/\sigma_X$ is called the **standardized version** of X. The mean and standard deviation of a standardized variable are always zero and one, respectively.

The **covariance** of X and Y is defined as $Cov(X, Y) = E[(X - \mu_X)(Y - \mu_Y)]$.

Properties of Covariance

$$Cov(a + bX, c + dY) = bdCov(X, Y) \tag{2.A.9}$$

$$Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y) \tag{2.A.10}$$

$$Cov(X + Y, Z) = Cov(X, Z) + Cov(Y, Z) \tag{2.A.11}$$

$$Cov(X, X) = Var(X) \tag{2.A.12}$$

$$Cov(X, Y) = Cov(Y, X) \tag{2.A.13}$$

If X and Y are independent,

$$Cov(X, Y) = 0 \tag{2.A.14}$$

The **correlation coefficient** of X and Y, denoted by $Corr(X, Y)$ or ρ, is defined as

$$\rho = Corr(X, Y) = \frac{Cov(X, Y)}{\sqrt{Var(X)Var(Y)}}$$

Alternatively, if X^* is a standardized X and Y^* is a standardized Y, then $\rho = E(X^*Y^*)$.

Properties of Correlation

$$-1 \le Corr(X, Y) \le 1 \tag{2.A.15}$$

$$Corr(a + bX, c + dY) = sign(bd)Corr(X, Y)$$

$$\text{where } sign(bd) = \begin{cases} 1 \text{ if } bd > 0 \\ 0 \text{ if } bd = 0 \\ -1 \text{ if } bd < 0 \end{cases} \tag{2.A.16}$$

$Corr(X, Y) = \pm 1$ if and only if there are constants a and b such that $Pr(Y = a + bX) = 1$.

<u>More Useful Properties</u>:

$$var(X - Y) = var(X) + var(Y) - 2\,cov(X, Y)$$

$$cov(W + X, Y + Z) = cov(W, Y) + cov(W, Z) + cov(X, Y) + cov(X, Z)$$

$$cov(aW + bX, cY + dZ) = (ac)\,cov(W, Y) + (ad)\,cov(W, Z) + (bc)\,cov(X, Y) + (bd)\,cov(X, Z)$$

$$cov(X, Y) = E(XY) - E(X)E(Y)$$

<u>If</u> X and Y are independent, then

$$E(XY) = E(X)E(Y)$$