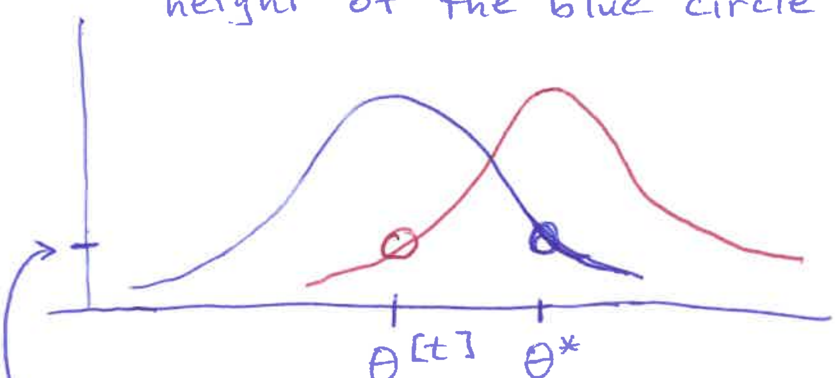
What it means for a proposal distribution to be symmetric.

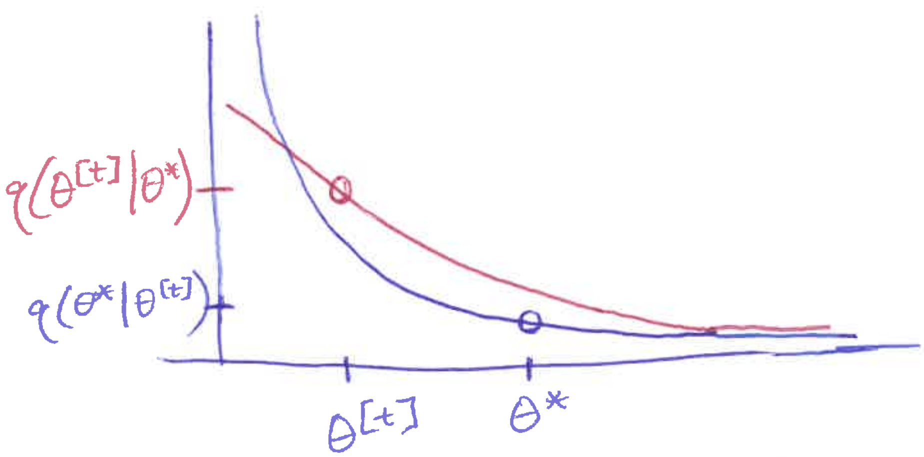
- Recall that $q(\cdot|\cdot)$ is our notation for the proposal density in the Metropolis-Hastings algorithm.
- If $\theta^{[t]}$ is the current value of the chain, and θ^* is a candidate value from the proposal distribution, then $q(\theta^*|\theta^{[t]})$ is the value (height) of the proposal density at argument value (location) θ^*.
- Let's look at an example in which the proposal distribution is <u>symmetric</u>. Suppose the proposal distribution is a <u>normal</u>, centered at the current value of the chain.
- So $q(\theta^*|\theta^{[t]})$ is the value of a normal density (that has mean $\theta^{[t]}$) at location θ^*: This is the height of the blue circle on the plot below:



$q(\theta^*|\theta^{[t]})$ is the same as $q(\theta^{[t]}|\theta^*)$ here

- Also, $q(\theta^{[t]}|\theta^*)$ is the value of a normal density (that has mean θ^*) at location $\theta^{[t]}$. This is the height of the red circle on that same plot

- Note that with this type of symmetric proposal density, $q(\theta^* | \theta^{[t]}) = q(\theta^{[t]} | \theta^*)$. This is what it means for a proposal distribution to be symmetric.
- In this case, the ratio $\frac{q(\theta^{[t]} | \theta^*)}{q(\theta^* | \theta^{[t]})} = 1$, so we don't need to include it as part of the "acceptance ratio".
- Now let's look at an example of a non-symmetric proposal distribution. Say, an exponential with mean equal to the current value of the chain.
- So $q(\theta^* | \theta^{[t]})$ is the value of an exponential density (that has mean $\theta^{[t]}$) at location θ^*. This is the height of the blue circle on the plot below:



- And $q(\theta^{[t]} | \theta^*)$ is the value of an exponential density that has mean θ^* at location $\theta^{[t]}$. This is the height of the red circle on that same plot.

- So you see in this <u>non-symmetric</u> case, $q(\theta^*|\theta^{[t]}) \neq q(\theta^{[t]}|\theta^*)$ and the ratio $\dfrac{q(\theta^{[t]}|\theta^*)}{q(\theta^*|\theta^{[t]})} \neq 1$. So here we need to include that ratio as part of the "acceptance ratio".

- In problem #2c on Homework 3, you are asked to argue that a proposal distribution is symmetric. this distribution in #2c is more complicated than in these two examples. Do your best to think through it (pick specific numbers for ρ^* and $\rho^{[t]}$ to help see what's going on) and come up with a reasonable argument, but don't worry too much if you can't fully understand this example. This one is challenging. Any reasonable thoughts will be given a good grade on this problem.