1) Suppose that in a random sample of 10 SEC games, the scores (points per game) of the Gamecocks’ men’s basketball team were recorded. The 10 observed scores were: 67, 79, 70, 76, 62, 66, 82, 63, 70, 53. Provide a graph of the empirical distribution function.

R code:
```r
scores <- c(67, 79, 70, 76, 62, 66, 82, 63, 70, 53)
plot(ecdf(scores), verticals=T)
```

2) Suppose 12 randomly selected students took an exam. Their scores were 81, 85, 89, 90, 90, 98, 76, 89, 82, 94, 72, 85. Provide a graph of the empirical distribution function.

R code:
```r
exams = c(81, 85, 89, 90, 90, 98, 76, 89, 82, 94, 72, 85)
plot(ecdf(exams), verticals=T)
```
3) Describe how you would get a point estimate for $F(Y \leq c)$ for a given number $c$, based on a random sample $X_1, \ldots, X_n$ which has the same distribution as $Y$. That is, if $X_1, \ldots, X_n$ is a random sample with the c.d.f. $F(x)$, then describe how you would estimate $F(c)$. Using your method, estimate the probability that the basketball team in problem 1 scores more than 75 points in its next SEC game.

The estimate of $F(c)$ is the value of the c.d.f. at location $c$, which is specifically obtained by counting the number of sample observations less than or equal to $c$ and then dividing that by $n$. The estimated probability that the basketball team in problem 1 scores more than 75 points in its next SEC game is $1 - \hat{F}(75) = 1 - 7/10 = 0.3$.

4) A random sample of 50 seniors at a high school were given a skills test. The average score of the sampled students was 80 and the standard deviation was 7. Find a 95% confidence interval for the mean score of all seniors at that high school.

A large-sample approximate 95% CI for the mean score of all seniors is $(80 - 1.96 \times 7 / \sqrt{50}, 80 + 1.96 \times 7 / \sqrt{50}) \rightarrow (78.06, 81.94)$
5) For a coin, let $p = P(\text{head})$ when the coin is tossed. The hypotheses are $H_0: p = 0.5$ and $H_1: p = 0.1$. The coin is tossed 4 times, and the critical region is "get one head or less".

(a) Find the significance level of the test.
(b) Find the power of the test.

\[ \alpha = P[\text{Reject } H_0 \mid H_0 \text{ true}] \]
\[ = P[T \leq 1 \mid T \sim \text{Bin}(4, 0.5)] \]

From Table A3, this is $0.3125$ where $T = \# \text{ heads out of } n=4$ tosses.

(b) Power = $P[\text{Reject } H_0 \mid H_0 \text{ false}]$
\[ = P[T \leq 1 \mid T \sim \text{Bin}(4, 0.1)] \]
From Table A3 with $n=4$, $p=0.5$, this is $0.9477$. 
6) A student is given three multiple-choice questions with five possible responses to each. If the student has studied the subject, he has an 80% chance of answering correctly on each question. If the student has not studied the subject (the null hypothesis) he has equal chances of responding each of the five possible ways on each question. The null hypothesis is rejected if the student gets all three answers correct.

(a) Find the level of significance of the test.
(b) Find the power of the test.
(c) What assumption about the data must we make that has not been explicitly stated?
(d) If a student takes the test and gets two questions correct, what is the p-value for that student?

(a) Note we have \( H_0 : p = 0.2 \) vs. \( H_1 : p = 0.8 \)

Let \( T \) = the number of correct answers out of the \( n = 3 \) questions.

Decision rule: Reject \( H_0 \) if \( T = 3 \)

\[
\alpha = P[\text{Reject } H_0 \mid H_0 \text{ true}] = P[T = 3 \mid p = 0.2] \\
= P[T \geq 3 \mid T \sim \text{Bin}(3, 0.2)] = 1 - P[T \leq 2 \mid T \sim \text{Bin}(3, 0.2)] \\
= 1 - 0.992 = 0.008 \text{ from Table A3}
\]

(b) Power = \( P[\text{Reject } H_0 \mid H_0 \text{ false}] = P[T \geq 3 \mid T \sim \text{Bin}(3, 0.8)] \\
= 1 - P[T \leq 2 \mid T \sim \text{Bin}(3, 0.8)] = 1 - 0.488 \text{ from Table A3} \\
= 0.512 \)

(c) We assume the responses on the 3 test questions are independent.

(d) \( P\)-value = \( P[T \geq t_{obs} \mid H_0 \text{ true}] \\
= P[T \geq 2 \mid T \sim \text{Bin}(3, 0.2)] = 1 - P[T \leq 1 \mid T \sim \text{Bin}(3, 0.2)] \\
= 1 - 0.896 = 0.104 \text{ from Table A3} \)
7) [Required for graduate students, extra credit for undergraduates] Assume the sample space contains 50 points, including points labeled A and B. Under the null hypothesis, all points in the sample space are equally likely. Under the alternative hypothesis, points A and B each carries 26 times as much probability as each of the other 48 points, which are still equally likely. The hypothesis test involves picking one sample point, and the critical region consists of points A and B.

(a) Find the level of significance.
(b) Find the power.

(a) \[ \alpha = \mathbb{P}[\text{Reject } H_0 \mid H_0 \text{ true}] = \mathbb{P}[\text{pick A or B} \mid H_0 \text{ true}] = \frac{2}{50} = 0.04 \]

(b) Power = \[ \mathbb{P}[\text{Reject } H_0 \mid H_0 \text{ false}] = \mathbb{P}[\text{pick A or B} \mid H_1 \text{ true}] = 0.52 \]

**Explanation:**
Under \( H_0 \): Each of the 50 sample points has probability \( \frac{1}{50} \).

Under \( H_1 \), \( P(A) = P(B) = 0.26 \), while each other sample point has probability 0.01.