STAT 521 – Spring 2019 – HW 3

Note: You must show your work wherever possible!

1. Suppose that on each play of a game, a gambler either wins 1 dollar with probability p or loses 1 dollar with probability $1 - p$. The gambler continues betting until he has cumulatively won n dollars or cumulatively lost m dollars. What is the probability that the gambler finishes the game a winner (i.e., having cumulatively won n dollars)?

2. For the Markov chain with states labeled $\{1, 2, 3, 4\}$ whose transition probability matrix is given below, find f_{i3} and s_{i3} for $i = 1, 2, 3$.

$$P = \begin{bmatrix} 0.4 & 0.2 & 0.1 & 0.3 \\ 0.1 & 0.5 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.2 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Consider a branching prcess with $\mu < 1$. Show that if $X_0 = 1$, then the expected number of individuals that ever exist in this population is given by $1/(1 - \mu)$. What is the expected number of individuals that ever exist in this population if $X_0 = n$?

4. For a branching process, calculate the extinction probability π_0 when

   (a) $P_0 = 1/4$, $P_2 = 3/4$.

   (b) $P_0 = 1/4$, $P_1 = 1/2$, $P_2 = 1/4$

   (c) $P_0 = 1/6$, $P_1 = 1/2$, $P_2 = 1/3$

5. Consider a Markov chain with state space $\{0, 1, 2, 3\}$ having the following transition probability matrix:

$$P = \begin{bmatrix} 0.4 & 0.2 & 0.1 & 0.3 \\ 0.2 & 0.2 & 0.1 & 0.5 \\ 0.3 & 0.2 & 0.5 & 0 \\ 0.2 & 0.1 & 0.4 & 0.3 \end{bmatrix}$$

   (a) If $X_0 = 0$, find the probability that state 2 is entered before state 3. [Hint: Let P_i = the probability of entering state 2 before entering state 3, starting from state i. Condition on the value of X_1.]

   (b) If $X_0 = 0$, find the mean number of transitions that occur before either state 2 or state 3 is entered. [Hint: First create a Markov chain with an absorbing state, and work with that.]