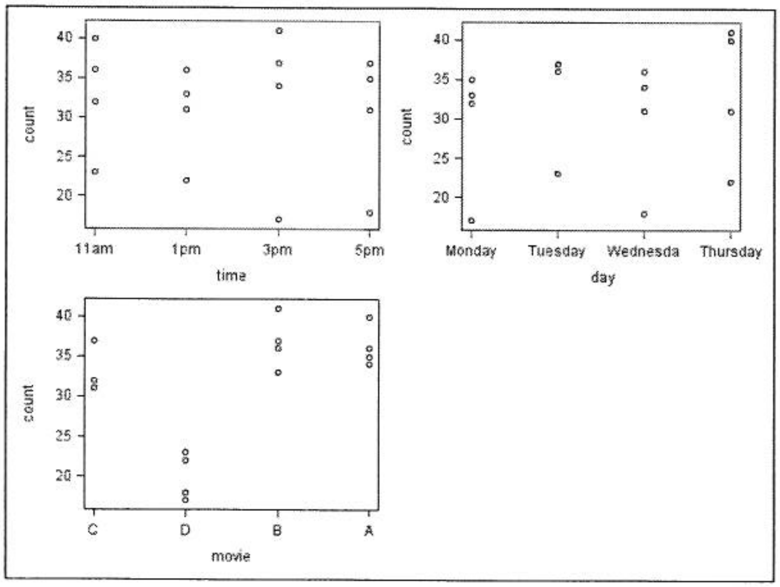
Day 2, Problem 4 Solution



1 Latin squares: Movie preference

The problem states that the outcomes are are counts out of 50; technically, a binomial regression model is arguably more appropriate. However, the counts are all away from 0 and 50, and a normal-errors model fits fine; the normal-errors analysis follows. Clearly, the movie is the treatment and movie time and day are blocking effects. Although the problem doesn't say so, a Latin Squares design works well here by completing the experiment in one week on one screen.

The scatterplots show a weak trend toward favoring movies as the week wears on. Movie D is clearly less favored than movies A, B, and C. Formally, we fit the model

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + e_{ijk},$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are movie (treatment) effects, $\beta_1, \beta_2, \beta_3, \beta_4$ are day of the week (blocking) effects, and $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ are time of day (blocking) effects. All of these respective main effects sum to zero for identifiability. In a Latin squares design, not all combinations of ijk are observed.

```
                                   The GLM Procedure

                                Class Level Information

                Class         Levels     Values
                time               4     11am 1pm 3pm 5pm
                day                4     Monday Thursday Tuesday Wednesda
                movie              4     A B C D

                        Number of Observations Read          16
                        Number of Observations Used          16
Dependent Variable: count

                                                Sum of
    Source                       DF            Squares     Mean Square    F Value    Pr > F
    Model                         9        815.0625000      90.5625000      21.84    0.0006
    Error                         6         24.8750000       4.1458333
    Corrected Total              15        839.9375000

                  R-Square     Coeff Var      Root MSE     count Mean
                  0.970385      6.476762      2.036132       31.43750

    Source                       DF          Type I SS     Mean Square    F Value    Pr > F
    time                          3         18.6875000       6.2291667       1.50    0.3066
    day                           3         60.6875000      20.2291667       4.88    0.0475
    movie                         3        735.6875000     245.2291667      59.15    <.0001


    Source                       DF        Type III SS     Mean Square    F Value    Pr > F
```

```
time                            3       18.6875000       6.2291667      1.50    0.3066
day                             3       60.6875000      20.2291667      4.88    0.0475
movie                           3      735.6875000     245.2291667     59.15    <.0001

                    Tukey's Studentized Range (HSD) Test for count

NOTE: This test controls the Type I experimentwise error rate, but it generally has a higher
                      Type II error rate than REGWQ.

                    Alpha                                        0.05
                    Error Degrees of Freedom                        6
                    Error Mean Square                        4.145833
                    Critical Value of Studentized Range       4.89560
                    Minimum Significant Difference              4.984

            Means with the same letter are not significantly different.

            Tukey Grouping         Mean       N    movie
                         A       36.750       4    B
                         A
                         A       36.250       4    A
                         A
                         A       32.750       4    C

                         B       20.000       4    D
```

We do not reject $H_0 : \gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$; time of day is not an effective blocking variable. We do reject $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$; blocking on day of the week effectively reduces variance. Finally, we reject that there is no differences among movies $H_0 : \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$. We wish to compare the movies. When comparing all possible combinations, Tukey has the highest power while controlling the familywise Type I error rate. We see that movies A, B, and C are not significantly different from each other, but movie D is significantly less favored compared to A, B, and C. If we add `cldiff` to the `means` statement we get the following:

```
                      Difference
         movie          Between      Simultaneous 95%
      Comparison          Means     Confidence Limits

         A - B           -0.500      -5.484     4.484
         A - C            3.500      -1.484     8.484
         A - D           16.250      11.266    21.234  ***
         B - C            4.000      -0.984     8.984
         B - D           16.750      11.766    21.734  ***
         C - D           12.750       7.766    17.734  ***
```

On average, people are likely to recommend 8 to 18 more people for movie C than D, 12 to 22 more people for B than D, and 11 to 21 more people for A than D. Note that the same conclusions are reached when one uses `lsmeans movie / adjust=tukey pdiff`.

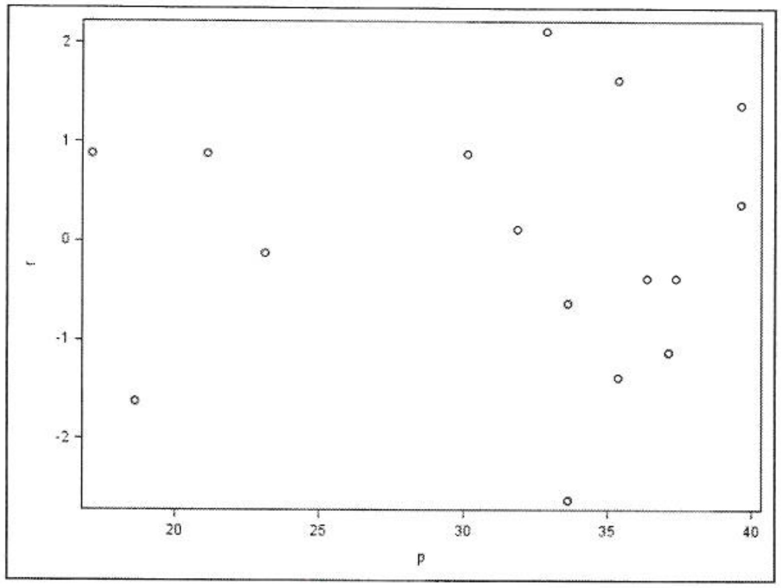
The studentized residuals versus predicted values and normal QQ plot show modeling assumptions okay.

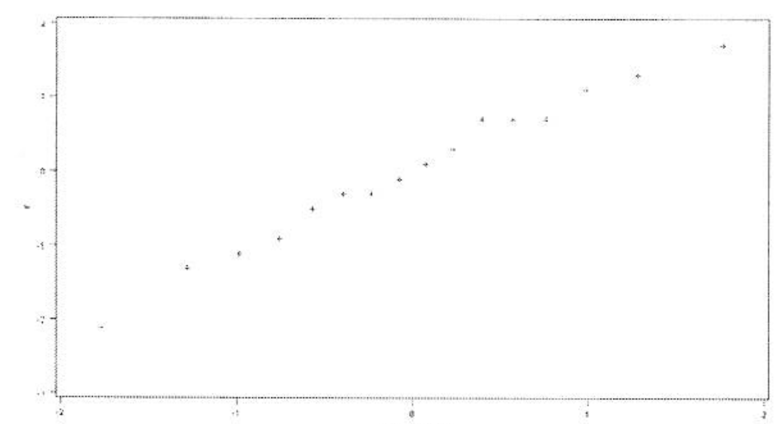
```
data movie;
input time$ day$ movie$ count;
datalines;
 11am Monday    C 32
 11am Tuesday   D 23
 11am Wednesday B 36
 11am Thursday  A 40
  1pm Monday    B 33
  1pm Tuesday   A 36
  1pm Wednesday C 31
  1pm Thursday  D 22
  3pm Monday    D 17
  3pm Tuesday   C 37
  3pm Wednesday A 34
  3pm Thursday  B 41
  5pm Monday    A 35
  5pm Tuesday   B 37
  5pm Wednesday D 18
  5pm Thursday  C 31
;
ods jpg; ods graphics on;
proc sgscatter;
 plot count*(time day movie); run;
ods graphics off; ods jpg close;


proc glm;
 class time day movie;
```

D2, P4

(3)





D2, P4


```
 model count=time day movie;
 means movie / tukey cldiff;
 output out=out p=p student=r; * studentizede residuals, not deleted;
run;

PROC UNIVARIATE noprint DATA=out;
  QQPLOT r / normal;
RUN;

ods jpg; ods graphics on;
proc sgscatter; plot r*p; run;
ods graphics off; ods jpg close;
```

Day 2, Problem 5 Solution

(a) $\underset{\sim}{b} = (X'X)^{-1}X'\underset{\sim}{Y}$

$$E(\underset{\sim}{b}) = (X'X)^{-1}X'E(\underset{\sim}{Y}) = (X'X)^{-1}X'(X\underset{\sim}{\beta} + \underset{\sim}{\Delta})$$

$$= (X'X)^{-1}X'X\underset{\sim}{\beta} + (X'X)^{-1}X'\underset{\sim}{\Delta}$$

$$= \underset{\sim}{\beta} + (X'X)^{-1}X'\underset{\sim}{\Delta} \neq \underset{\sim}{\beta} \quad \text{in general.}$$

(b) Bias vector $= (X'X)^{-1}X'\underset{\sim}{\Delta}$

Note $X'X = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ X_{11} & X_{21} & \cdots & X_{n1} \\ X_{12} & X_{22} & \cdots & X_{n2} \end{bmatrix} \begin{bmatrix} 1 & X_{11} & X_{12} \\ 1 & X_{21} & X_{22} \\ \vdots & \vdots & \vdots \\ 1 & X_{n1} & X_{n2} \end{bmatrix}$

$$= \begin{bmatrix} n & \sum X_{i1} & \sum X_{i2} \\ \sum X_{i1} & \sum X_{i1}^2 & \sum X_{i1}X_{i2} \\ \sum X_{i2} & \sum X_{i1}X_{i2} & \sum X_{i2}^2 \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 120 & 15 \\ 120 & 1240 & 170 \\ 15 & 170 & 25 \end{bmatrix} \quad \text{for these predictor values.}$$

$$\Rightarrow (X'X)^{-1} = \begin{bmatrix} 0.467 & -0.1 & 0.4 \\ -0.1 & 0.033 & -0.167 \\ 0.4 & -0.167 & 0.933 \end{bmatrix}$$

and $X'\underset{\sim}{\Delta} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 15 \\ 0 & 0 & \cdots & 2 \end{bmatrix} \begin{bmatrix} \delta \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \delta \\ \delta \\ 0 \end{bmatrix}$

$$\Rightarrow (X'X)^{-1}X'\underset{\sim}{\Delta} = \begin{bmatrix} 0.367\,\delta \\ -0.067\,\delta \\ 0.233\,\delta \end{bmatrix}$$

Day 2, Problem 5 Solution

(c) Several solutions are possible, but the simplest approach is to define the variable

$$X_{i3} = \begin{cases} 1 & \text{if } i = 1 \\ 0 & \text{if } i = 2, 3, \ldots, 15 \end{cases}$$

Fit the model

$$E(Y) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3$$

and do the usual t-test of

$H_0: \beta_3 = 0$ vs. $H_a: \beta_3 \neq 0$ using

$$t^* = \frac{b_3}{s\{b_3\}}$$

Since $E(Y) = (\beta_0 + \beta_3) + \beta_1 X_1 + \beta_2 X_2$ if $i = 1$

and $E(Y) = \beta_0 + \beta_1 X_1 + \beta_2 X_2$ if $i \in \{2, 3, \ldots, 15\}$,

this is equivalent to testing $H_0: \delta = 0$ vs. $H_a: \delta \neq 0$.

(d) From R, $t^* = 2.86$ with a P-value of 0.0155.

So we reject H_0 and conclude $\delta \neq 0$.

A point estimate for δ is $b_3 = 7.2681$.

Day 2, Problem 6 Solution

a) Let Y represent the true underlying weight:

$Y \sim N(26.1,\ 0.05^2)$

$X = 25.9 \iff 25.85 \le Y < 25.95$

$P(25.85 \le Y < 25.95) = P(-5 \le Z < -3) = .00135$

$X = 26.0 \iff 25.95 \le Y < 26.05$

$P(25.95 \le Y < 26.05) = P(-3 \le Z < -1) = .1573$

$X = 26.1 \iff 26.05 \le Y < 26.15$

$P(26.05 \le Y < 26.15) = .6827$

$X = 26.2 \iff 26.15 \le Y < 26.25$

$P(26.15 \le Y < 26.25) = .1573$

$X = 26.3 \iff 26.25 \le Y < 26.35$

$P(26.25 \le Y < 26.35) = .00135$

Prob. Distn. of X:

X	P(x)
25.9	.00135
26.0	.1573
26.1	.6827
26.2	.1573
26.3	.00135

All other values of X have probability 0 to 3 decimal places.

(b) Sampling Distribution of $\bar{X}$:

$\bar{X}$	$P(\bar{X})$	
25.9	$(.00135)^2$	≈ 0
25.95	$2(.00135)(.1573)$	$\approx .00042$
26.0	$(.1573)^2 + 2(.00135)(.6827)$	$\approx .02659$
26.05	$2(.00135)(.1573) + 2(.1537)(.6827)$	$\approx .2152$
26.1	$(.6827)^2 + 2(.1573)^2 + 2(.00135)^2$	$\approx .51557$
26.15	$2(.00135)(.1573) + 2(.1537)(.6827)$	$\approx .2152$
26.2	$(.1573)^2 + 2(.00135)(.6827)$	$\approx .02659$
26.25	$2(.00135)(.1573)$	$\approx .00042$
26.3	$(.00135)^2$	≈ 0

(c) $\text{Power} = P[\text{Reject } H_0] = P[Z > 1.645]$

$$= P\left[\frac{\bar{X} - \mu_0}{(.05)/\sqrt{2}} > 1.645\right] = P\left[\bar{X} - \mu_0 > .05816\right]$$

$$= P[\bar{X} > \mu_0 + .05816]$$

Note $25.95 - .05816 = 25.89184$. Hence:

For $\mu_0 < 25.89184$, Power ≈ 1

Else for $\mu_0 < 25.94184$, Power $\approx 1 - .00042 = .99958$

Else for $\mu_0 < 25.99184$, Power $\approx .99958 - .02659 = .97299$

Else for $\mu_0 < 26.04184$, Power $\approx .75779$

Else for $\mu_0 < 26.09184$, Power $\approx .24222$

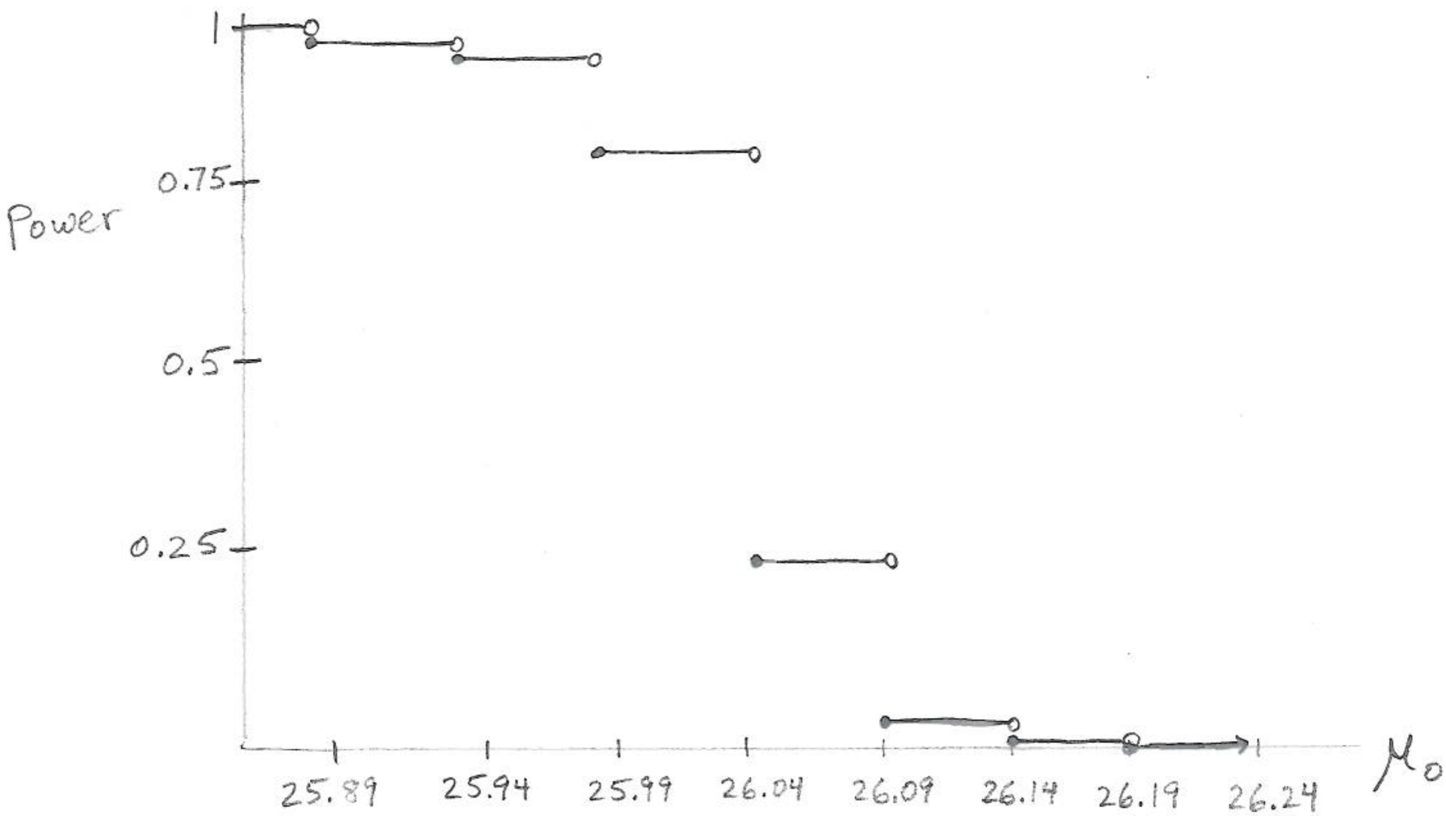
Else for $\mu_0 < 26.14184$, Power $\approx .02702$

Else for $\mu_0 < 26.19184$, Power $\approx .00043$

Else for $\mu_0 > 26.24184$, Power ≈ 0.

Day 2, Problem 6 Solution (continued)

Sketch of Power Function:



Actual significance level = Power at $\mu_0 = 26.1$

$\approx .02702$ or $.02701$.

(d)


```
my.Xs <-
round(cbind(rnorm(n=10000,mean=26.1,sd=0.05),rnorm(n=10000,mean=26.1,sd=0.05)), 1)

my.Xbars <- apply(my.Xs, 1, mean)

my.test.stats <- (my.Xbars - 26.1)/(0.05/sqrt(2))

my.TypeI.error.rate <- mean(my.test.stats > 1.645)

print(my.TypeI.error.rate)

### Empirical Type I error rate should be somewhere around 0.027.
```

(very similar to calculated "actual" significance level)