Specific Comparisons

- **If any of the F-tests reveal that the factor(s) have significant effects on the response, we can perform:**
  - **Preplanned comparisons (contrasts)**
  - **Post-hoc multiple comparisons (Fisher LSD or Tukey)**

in order to determine which factor levels produce significantly different mean responses.

- **This is straightforward when there is no significant interaction between factors.**

- **We may then treat each factor separately, and use contrasts or multiple comparisons to compare mean responses among the levels of each factor.**

- **Basically just like in previous chapter, except we do it for two factors separately.**

Example:

● If we do have significant interaction (as we actually did in the gas mileage example), we must investigate contrasts about one factor <u>given a specific level of the other factor</u>.

<u>Example 1</u>: Do the mean mileages of 4-cylinder and 6-cylinder engines differ significantly, when the oil type is "Gasmiser"?

Relevant contrast:

We test:

<u>Example 2</u>: Do the mean mileages for the cheap oil ("standard") and the expensive oils differ significantly, when the engine is "4-cylinder"?

Relevant contrast:

We test:

Conclusions based on computer output:

Post-Hoc Comparisons

● **If there is significant interaction, we test for significant differences in mean response for each pair of factor level combinations.**

We test:

● **Again, Fisher LSD procedure has P{Type I error} = α for each comparison.**

● **Tukey procedure has P{at least one Type I error} = α for the entire set of comparisons.**

● **For Tukey procedure, we conclude a difference in mean response is significant, at level α, if:**

(for $i' \neq i''$, $j' \neq j''$)

Example (Gas mileage data):

Additional Considerations

- **What if we have no replication (i.e., $n = 1 \rightarrow$ one observation for each cell)?**

- **We then have no estimate of σ^2 (the variation among responses in the same cell).**

- **Solution: Assume there is no interaction. The interaction MS will then serve as an estimate of σ^2.**

- **If we do this, and interaction does exist, then our F-tests will be biased (conservative $\rightarrow$ less likely to reject H_0).**

<u>Three or More Factors</u>

● If we have three or more factors, we have the possibility of <u>higher-order interactions</u>.

<u>Example</u>: Factors A, B, and C:

● If the 3-way interaction is significant, this implies, for example, that the A$\times$B interaction is not consistent across the levels of C.

● Having 3 or more factors means having lots of "cells".

● If resources are limited, the number of replicates could be small (n = 1? n = 2?)

● It may be better to assume higher-order interactions do not exist (often they are of no practical interest anyway).

● Thus we could devote more degrees of freedom to estimating σ^2.

● Analysis of three-factor studies can be done with software in a similar way.

Example: (Table 9.27 data, p. 515)

Response: Rice yield

Factors: Location (4 levels)
Variety (3 levels)
Nitrogen (4 levels)

● **We have *n* = 1 observation for each factor level combination.**

Analysis: