Design of Experiments

- Factorial experiments require a lot of resources
- Sometimes real-world practical considerations require us to design experiments in specialized ways.
- The design of an experiment is the specification of how treatments are assigned to experimental units.

**Goal:** Gain maximum amount of reliable information using minimum amount of resources.

- Reliability of information is measured by the standard error of an estimate.
- How to decrease standard errors and thereby increase reliability?

- Recall the One-Way ANOVA:
- Experiments we studied used the Completely Randomized Design (CRD).
The estimate of $\sigma^2$ was MSW. This measured the variation among responses for units that were treated alike (measured variation within groups).

We call this estimating the experimental error variation.

What if we divide the units into subgroups (called blocks) such that units within each subgroup were similar in some way?

We would expect the variation in response values among units treated alike within each block to be relatively small.

**Randomized Block Design (RBD)**

RBD: A design in which experimental units are divided into subgroups called blocks and treatments are randomly assigned to units within each block.

Blocks should be chosen so that units within a block are similar in some way.

Reasons for the variation in our data values:

CRD ................................ RBD
- Benefits of a reduction in experimental error:
  - decreases MSW (denominator of F* ratios used in F-tests) → more power to reject null hypotheses
  - decreases standard errors of means → shorter CIs for mean responses

**Example 1:** Suppose we investigate whether the average math-test scores of students from 8 different majors differ across majors.
- But … students will be taught by different instructors.
- We’re not as interested in the instructor effect, but we know it adds another layer of variability.

Solution:

**Example 2:** Lab animals of a certain species are given different diets to determine the effect of diet on weight gain.
- Possible block design:
Example 3: An industrial experiment is conducted over several days (with a different lab technician each day).

- Possible block design:

Example 4: (Table 10.2 data)

\[ Y = \text{wheat crop yield} \]

- experimental units = plots of wheat
- treatments = 3 different varieties of wheat
- blocks = regions of field

Possible arrangement:
• The data are given in Table 10.2.

• Note: Variety A has the greatest mean yield, but there is a sizable variation among blocks.

• If we had used a CRD, this variation would all be experimental error variance (inflates MSW).

• Analysis as CRD (ignoring blocks):

• But … within each block, Variety A clearly has the greatest yield (RBD will account for this).
Formal Linear Model for RBD

- This assumes one observation per treatment-block combination.

\[ Y_{ij} = \text{response value for treatment } i \text{ in block } j \]
\[ \mu = \text{an overall mean response} \]
\[ \tau_i = \text{effect of treatment } i \]
\[ \beta_j = \text{effect of treatment } j \]
\[ \varepsilon_{ij} = \text{random error term} \]

- Looks similar to two-factor factorial model with one observation per cell.

**Key difference:** With RBD, we are not equally interested in both factors.
- The treatment factor is of primary importance; the blocking factor is included merely to reduce experimental error variance.

- With RBD, the block effects are often considered random (not fixed) effects.

- This is true if the blocks used are a random sample from a large population of possible blocks.
● If treatment effects are fixed and block effects are random, the RBD model is called a **mixed model**.

● In this case, the treatment-block interaction is also random.

● This interaction measures the variation among treatment effects across the various blocks.

● The mean square for interaction is used here as an estimate of the **experimental error variance** $\sigma^2$.

**Expected Mean Squares in RBD**

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>E(MS)</th>
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</table>

• Testing for an effect on the mean response among treatments:

\[ H_0: \]

• The correct test statistic is apparent based on \( \text{E(MS)} \):

\[ F^* = \quad \text{Reject } H_0 \text{ if:} \]

• Testing for significant variation across blocks:

\[ H_0: \]

• The correct test statistic is again apparent:

\[ F^* = \quad \text{Reject } H_0 \text{ if:} \]

**Example:** (Wheat data – Table 10.2)

• The ANOVA table formulas are the same as for the two-way ANOVA.

• We use software for the ANOVA table computations.
RBD analysis (Wheat data):

\( F^* = \)

- We conclude that the mean yields are significantly different for the different varieties of wheat. At \( \alpha = 0.05 \), we reject \( H_0: \tau_1 = \tau_2 = \tau_3 = 0 \).

**Note** (for testing about blocks):

\( F^* = \)

- We would also reject \( H_0: \sigma_\beta^2 = 0 \) and conclude there is significant variation among block effects.

- We can again make pre-planned comparisons using contrasts.

**Example:** Is Variety A superior to the other two varieties in terms of mean yield?

\( H_0: \)

\( H_a: \)

Result: