Further Investigation of Factor Effects

Case I: No Interaction

• Many of the types of inference we may make are similar to the single-factor analysis. We can obtain:

CI for a Factor Level Population Mean:

• For any level *i* of A, SAS will give a CI for $\mu_{i\cdot}$

• For any level *j* of B, SAS will give a CI for $\mu_{\cdot j}$

SAS Example (Bakery data):

CL option to LSMEANS statement:

CI and Test about a Contrast of Factor Level Means

• If we are interested in a contrast among the level means of

factor A ($L =$)

or of factor B ($L =$), SAS will provide CI and hypothesis test results.

SAS example (Bakery data): Suppose contrast of interest is

Interpretation?

95% CI for L:

Test of

Multiple Pairwise Comparisons of Factor Level Means

• Tukey's Procedure will provide simultaneous CIs for

and all simultaneous tests of

• Similarly, the Tukey procedure gives all simultaneous CIs for

and simultaneous tests of

SAS Example (Bakery data):
We earlier found a significant difference in mean sales among the levels of Height. Which particular levels are significantly different? (Use family significance level 0.05.)

Tukey procedure in SAS:

• Depending on the comparison(s) of interest, the Bonferroni or Scheffe procedure could be used instead.

Case II: Significant Interaction Present

• When interaction is present, we must compare mean responses at each level of both factors.

• That is, we do not compare

but we compare each μ_{ij} separately.

Example (Melon data):

Response = Percent of Melon Plants Surviving
Factor A = Fungicide Type (Levels: B, T, C)
Factor B = Concentration of Fungicide (Levels: 100, 1000 ppm)

• The ANOVA table shows a significant Fungicide × Concentration Interaction (P-value =) at the 0.05 level.

• We may compare all possible pairs of treatment means simultaneously using Tukey's procedure.

SAS Example:

<u>Of interest</u>: Is the difference between Fungicides B and C the same regardless of the level of concentration?

• Be CAREFUL to note how SAS orders the levels of each factor! SAS orders the Fungicide Levels:

Contrast of Interest:

We must write this in terms of the factor effects to properly specify the ESTIMATE statement:

SAS Example of ESTIMATE statement to perform the test and CI about *L*:

Pooling Sums of Squares in the Two-Factor ANOVA

• The typical approach to testing in the Two-Way ANOVA is to treat this model:

as the full model (assuming that model assumptions are met), regardless of the conclusions of any of the formal tests.

• Some statisticians suggest that if the F-test for interaction effects has concluded that there are no significant interactions, the full model for testing for main effect of factors A and B can be the revised full model:

• This will not affect SSA nor SSB, but the revision <u>does</u> affect the denominator SS.
• The "new" error SS will be the sum of SSAB and SSE from the original full model.
• Similarly, the "new" error df will be the sum of df(AB) and df(Error) from the original full model.

• This is called "pooling" the interaction and error SS (and df).

Example: (Castle Bakery data)

• This "pooling" affects the power and significance level of the tests for the main effects of A and B.
• It can improve power, especially when the original error df are small and the interaction df are somewhat large.

• But it should be done with caution, because it can produce biased tests of the main effects if the interaction effects are not truly equal to zero.

• Recommendation: Only consider pooling the SS when:
(1)

and

(2)

Power in the Two-Way ANOVA

• In SAS, the GLMPOWER procedure will calculate power for the F-tests for interaction and main effects in the Two-way ANOVA.

• The user is required to specify the arrangement of hypothetical treatment population means for which the power is desired, as well as a "guess" for σ, the standard deviation of the random errors.

• See example on course web page.

Situation with Only One Observation per Treatment (One Observation per Cell)

• In this case, variability within treatments (which is typically measured by SSE and MSE) cannot be estimated (if there's only one observation per cell, SSE is automatically).

• Hence we have no estimator of σ^2.

• If there is no interaction between A and B, we can let SSAB play the role of SSE, and in this case MSAB will be an unbiased estimate of σ^2.

Note: In the no-interaction case,
is a better estimate of μ_{ij} than is the

• If we use the no-interaction model when A×B interaction actually <u>does</u> exist, our CIs will be too wide and our tests will have less power to detect truly significant effects.

• The "Tukey Test for Additivity" can, in this situation, test for the specific type of interaction

• The test statistic F* (see pg. 887) has

• We can use Tukey's Additivity Test to informally check for general interactions.

<u>Note</u>: If interaction <u>is</u> present, we can try a transformation of *Y* to remove it, or use advanced methods (see pg. 889 ref.)

<u>Example</u>: (Insurance data)

Response = Premium (in dollars)
Factor A = Size of City (Levels: Small, Medium, Large)
Factor B = Region (Levels: East, West)

• Only one observation in each of the 6 cells (treatments)
→

From SAS: