Chapter 27: Repeated Measures Designs

- Occur when several observations are taken (over time) on the same subject.

- For a group of \( s \) subjects, each subject is given a sequence of \( r \) treatments.

- Because observations on the same subject are likely to be similar, the subjects play the role of

**Example 1:** In several cities, a fast-food chain produces four different advertising campaigns (given in January, March, May, July). The sequencing of the campaigns is randomly chosen in each city.

**Response:** Sales for that month  
**Subjects:**

**Example 2:** For a sample of sick patients, 5 different drugs are given in sequence over a period of time. The order of the drugs is randomly chosen for each patient.

**Response:** Improvement in white blood cell count  
**Subjects:**

- The analysis is identical to that of a RCBD, with the subjects serving as blocks.
- Typically, the subjects are a random sample.

**Model:**
As with a mixed model,

- The ANOVA and tests are identical in this model to the RCBD analyses.

- If two observations near in time within a subject have a different correlation than two observations far apart in time within a subject, then this compound-symmetry assumption is not appropriate.

- More advanced methods must be used in that case (see the conservative test method given in Comment 2, pg. 1065).

- This compound-symmetry assumption can be examined by viewing the estimated within-subjects variance-covariance matrix, with entries:
Example (Wine data):
Checking model assumptions:

Inferences comparing the four wines:

• Section 27.3 discusses two-factor experiments with repeated measures on one of the factors.
Example (shoe data):  **Response:** Sales
**Factor A:** Type of Advertising Campaign
**Factor B:** Time (1 = before, 2 = during, 3 = after campaign)
**Subjects:** 10 test markets (chosen at random)

Note: Five of the test markets received campaign 1, and the other five received campaign 2 (subjects are “nested” within factor A – more about this later).

Note: If the data in such a study are unbalanced, the methods of Section 25.7 must be used (in SAS, use PROC MIXED in unbalanced case rather than PROC GLM).
Nonparametric Methods in ANOVA

• In ANOVA, sometimes the normality assumption for the response may not be reasonable (even after transformation?)
• Some rank-based distribution-free alternatives to the common ANOVA tests have been developed.

Kruskal-Wallis Test

• An alternative to the one-way ANOVA F-test:

Model:

• We assume the $r$ populations are continuous and identical (in shape, variance, etc.) except possibly for their centers.

• Procedure: rank the entire data set from 1 (smallest) to $n_T$ (largest), in ascending order of response values. (If there are tied values, midranks are used.)
• Replace the response values with their ranks and perform the ANOVA calculations on the ranks.

The Kruskal-Wallis test statistic is
• Our hypotheses are:

• For large samples (rule of thumb: )

Note: If the ranks inside one treatment vary greatly from the ranks inside other treatment(s):

• With small samples, tables/software are available for performing the K-W test based on the exact null distribution of $\chi^2_{KW}$.

**Example** (Soil data):
- **Response**: Percentage of clay in soil
- **Factor**: Location (4 different levels)
- Six observations were made in each location.

Boxplots show

SAS/R Results:
• Bonferroni procedure provides simultaneous rank-based testing limits for

• For our example:
**Friedman Test**

- A distribution-free test for treatment effects for a RCBD.

**Model:**

- We assume each treatment appears **once** within each block.

- Block effects could be random; in that case, $\rho_i$ and $\varepsilon_{ij}$ need **not** have a normal distribution, merely a continuous distribution.

**Hypotheses:**

**Procedure:** Rank all responses **within each block** in ascending order, from 1 (smallest) to $r$ (largest).

- Perform ANOVA calculations for RCBD on the **within-block ranks**.

The Friedman test statistic is:
For large samples (rule of thumb:)

For small samples, tables of critical values are available for exact tests based on $\chi^2_F$.

Example (Wind speed data):

Response: average wind speed reduction
Treatments: 5 different distances to shelterbelt (line of trees)
Blocks: 9 different months

• Is there a significant effect on mean wind speed reduction?

Note: When each treatment appears $d \geq 2$ times within each block, the Mack-Skillings test is an appropriate extension of Friedman’s test.

Note: Cochran’s test is a version of Friedman’s test for binary responses.

Note: When $r = 2$, the K-W test reduces to the ____________

When $r = 2$, the Friedman test reduces to the ____________