Expected Values

• Example 1: A campus organization is holding a raffle to raise money. There are two prizes: a $200 gift certificate to the campus bookstore, and a $50 gift certificate.

• 1000 raffle tickets will be sold (at $1 apiece), and two of the tickets will be winners.

• For a ticket buyer, what is the expected return?

• This would indicate what a buyer would consider a “fair price” (disregarding the philanthropic aspect!)
Calculating Expected Values

• We can easily calculate the expected value of a random phenomenon that has a finite number of possible numerical outcomes.

• First we must specify a probability model giving the probability of each outcome occurring.

• Step 1: Simply take each numerical outcome and multiply each one by its probability.

• Step 2: Then add up all those resulting products.

• Note: Each outcome is weighted by the likelihood of that outcome occurring.
Calculating Expected Values (continued)

• Recall Example 1: The possible outcomes are the different possible “winnings” on the raffle ticket purchase.

• So the outcomes are 200, 50, or 0.

• The corresponding probabilities for these outcomes are: $1/1000$, $1/1000$, and $998/1000$.

• So the expected winnings is: $200 \times 0.001 + 50 \times 0.001 + 0 \times 0.998 = 0.25$.

• Therefore a purchaser of one ticket has an “expected winnings” of $0.25$, or 25 cents.

• Clearly the $1$ price is “unfair” from the buyer’s perspective . . . but it’s for a good cause!
Interpreting Expected Values (continued)

• Note that the “expected value” may not in fact be one of the possible values of the variable.

• For the variable “raffle winnings,” the expected value was $0.25, but that wasn’t one of the possible values that the buyer could win.

• We can interpret the expected value as a *long-run average*.

• If the experiment were repeated many times, the *average value* of the variable across those repetitions would be near the expected value.

• If the buyer bought many tickets, she would win about $0.25 for each ticket purchased, *on average*. 
Clicker Quiz 1

In college football, after scoring a touchdown, a team may try for 1 point by kicking the ball through the goal posts. An unsuccessful kick attempt results in 0 points. Suppose teams are successful in such kicks 96% of the time. What is the expected point total from this type of kick attempt?

A. 1
B. 0
C. 0.96
D. 1.96
Clicker Quiz 2

In college football, after scoring a touchdown, a team may try for 2 points by running or passing the ball for a score. An unsuccessful run or pass attempt results in 0 points. Suppose teams are successful in these types of attempts 44% of the time. What is the expected point total from this type of attempt?

A. 0.44
B. 0.88
C. 2
D. 1
Another Expected Value Example

*Example 2*: A probability model for the number of vehicles owned in American households is the following (note that a negligible proportion have more than 5 vehicles):

<table>
<thead>
<tr>
<th>Number of vehicles</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion</td>
<td>0.10</td>
<td>0.34</td>
<td>0.39</td>
<td>0.13</td>
<td>0.03</td>
<td>0.01</td>
</tr>
</tbody>
</table>

The expected number of vehicles in a randomly selected American household is:

\[
0 \times 0.10 + 1 \times 0.34 + 2 \times 0.39 + 3 \times 0.13 + 4 \times 0.03 + 5 \times 0.01 = 1.68
\]
cars.
The Law of Large Numbers in the Real World

- The *law of large numbers* says that if a random phenomenon is repeated many times, the *sample mean* of these many outcomes will be close to the expected value of the phenomenon.

- This assurance guarantees that lotteries will make money by setting up the prize system so that the expected winnings for a ticket buyer is less than the price of the ticket.

- Casinos structure their games so that the expected profit of a gambler is a bit less than zero.

- When lots of gamblers play, some will win money . . . but in the long run, the casino knows it will come out ahead.
The LLN in the Real World (continued)

- Life insurance companies set up policies knowing the probability of having to pay a claim for a given customer – they set the price of the premium so that the company’s expected profit is positive.

- *Interesting question:* If the company has a positive expected profit from an insurance policy, the customer’s expected profit must be negative.

- Does it ever make sense for a customer to buy an insurance policy?

- *Think:* Does the law of large numbers apply to the customer in the same way as it does to the company?

- *Another example:* “Deal or No Deal” game show
Beating the Odds?

- Some gamblers believe they have a “system” that will allow them to earn a profit while gambling.
- In pure games of chance, this won’t work in the long run.
- For any game with an expected profit that is less than zero, the gambler will lose money in the long run.
Finding Expected Values with Simulation

- For simple probability models, we can find the expected value using simple math.
- With complicated models, the calculations can become very difficult.
- It’s often easier to estimate the expected value through simulation.
- We simply simulate the random phenomenon many times on a computer and keep track of the numerical outcome each time.
- The law of large numbers tells us that the average of these many outcomes will be very close to the true expected value (see example applet).