

STAT 520 – Homework 1B – Fall 2023

3) Suppose $\{e_t\}$ is a normal white noise process with mean zero and variance σ_e^2 . Let $\{Y_t\}$ be a process defined as:

$$Y_t = e_t + \theta e_{t-1}.$$

- a) Find the autocovariance function and autocorrelation function of Y_t for any general θ . [Hint: Calculate $\text{cov}(Y_t, Y_{t-k})$ case-by-case for several values of k .] Also, find the autocovariance function and autocorrelation function of Y_t if $\theta = 2$. Show all your steps clearly.
- b) Is the time series $\{Y_t\}$ stationary? Explain your answer.

4) Apply a moving average filter to Y_t , where Y_t is the natural logarithm of the Johnson and Johnson earnings data (the original data are given in the `jj` object in the `astsa` package).

Specifically, let

$$V_t = (Y_t + Y_{t-1} + Y_{t-2} + Y_{t-3}) / 4.$$

The R code `v = filter(y, rep(1/4, 4), sides = 1)`

may be helpful in implementing this. Type `help(filter)` in R for more details about this R function. Plot Y_t as a line and overlay (superimpose) V_t as a dashed line, and provide this plot. Discuss whether the moving average filter captures the overall trend in the time series.

5) [Required for graduate students, extra credit for undergraduate students] Suppose $\{e_t\}$ is a normal white noise process with mean zero and variance σ_e^2 . Let $\{Y_t\}$ be a process defined as: $Y_t = e_t e_{t-1}$. Showing all your steps, find the mean function and the autocovariance function of Y_t . Is the time series $\{Y_t\}$ stationary? Explain your answer.