STAT 520 – Homework 3A – Fall 2023

1) Consider the MA(2) process, where all the $\{e_t\}$ values are independent white noise with variance σ^2_e:
$Y_t = e_t - 0.5\, e_{t-1} - 0.3\, e_{t-2}$
a) Find $\text{cov}(Y_t, Y_t) = \text{var}(Y_t)$.
b) Find $\text{cov}(Y_t, Y_{t-1})$ and, from this, find the lag-1 autocorrelation $\text{corr}(Y_t, Y_{t-1})$.
c) Find $\text{cov}(Y_t, Y_{t-2})$ and, from this, find the lag-2 autocorrelation $\text{corr}(Y_t, Y_{t-2})$.
d) Argue that $\text{cov}(Y_t, Y_{t-k}) = 0$ for all $k \geq 3$.

2) **[Mandatory for graduate students, extra credit for undergraduates]**
Repeat all parts of Problem 1, but for the process:
$Y_t = e_t - 1.2\, e_{t-1} + 0.7\, e_{t-2}$

3) Consider the AR(1) process: $Y_t = \phi Y_{t-1} + e_t$
Show that if $|\phi| = 1$, the process cannot be stationary. [Hint: Take variances of both sides.]

4) The monthly U.S. air passenger miles flown for January 1960 to December 1971 are in the `airpass` object in the `TSA` package. Type `library(TSA); data(airpass); print(airpass)` in R to see the data set.
(a) Plot the time series, using plotting symbols that allow you to check for seasonality. What basic pattern do you see from the plot?
(b) Plot the (natural) log-transformed time series. What basic pattern do you see from the plot? What effect has the log transformation had?
(c) Plot the differences of the natural logarithms. Does this plot suggest that a stationary model might be appropriate for the differences of the natural logarithms? Briefly explain.
(d) Plot the fractional relative differences, $(Y_t - Y_{t-1})/Y_{t-1}$, which can be obtained in R with the code:

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diff(airpass)/(zlag(airpass)[-1])
```

How do these values compare with the differences of the natural logarithms from part (c)?