

STAT 520 – Homework 4 – Fall 2023

For Homework 4A, do Problem 1 and Problem 3 below.

For Homework 4B, do Problem 2 and Problem 4 below.

[For these homework assignments, all students should do all the assigned problems.]

Recall: The `arima` function in the `TSA` package in R fits a mean-centered model by default, if the specified model is an AR, MA or ARMA model (i.e., a stationary model).

1) A data set of 57 consecutive measurements from a machine tool are in the `deere3` object in the `TSA` package. Type `library(TSA); data(deere3); print(deere3)` in R to see the data set.

(a) Estimate the parameters of a (mean-centered) AR(1) model for this series. Use the least squares method and maximum likelihood, and report the estimated parameters from each of these methods. Comment on any similarities and differences.

(b) Estimate the parameters of a (mean-centered) AR(2) model for this series. Use the least squares method and maximum likelihood, and report the estimated parameters from each of these methods. Comment on any similarities and differences.

(c) Compare the results of the ML fits from parts (a) and (b) using AIC. Which model do you believe is preferable? Briefly explain your answer.

2) Consider the (mean-centered) AR(1) model for the `deere3` data in the `TSA` package, estimated using maximum likelihood.

(a) Give a basic plot of the standardized residuals over time and a Q-Q plot of the residuals. Comment on what these tell you about the adequacy of the model.

(b) Give a plot of the sample autocorrelation function of the residuals. Also perform a runs test and a Ljung-Box test (with $K = 8$). Comment on what these tell you about whether the errors are independent in this model.

(c) Diagnose the fit of the AR(1) model by using the overfitting strategy.

3) A data set of 324 measurements of an industrial robot's positions are in the `robot` object in the `TSA` package. Type `library(TSA); data(robot); print(robot)` in R to see the data set.

(a) Estimate the parameters of a (mean-centered) AR(1) model for these data, using maximum likelihood. Give the equation of the estimated model.

(b) Give an approximate 95% confidence interval for ϕ , the coefficient in the AR(1) model.

(c) Estimate the parameters of an IMA(1,1) model (a.k.a. an ARIMA(0,1,1) model) for these data, using maximum likelihood. Give the equation of the estimated model.

(d) Compare the results from parts (a) and (c) using AIC.

4) Consider the (mean-centered) AR(1) model for the `robot` data in the `TSA` package, estimated using maximum likelihood.

(a) Give a basic plot of the standardized residuals over time and a Q-Q plot of the residuals. Comment on what these tell you about the adequacy of the model.

(b) Give a plot of the sample autocorrelation function of the residuals. Also perform a runs test and a Ljung-Box test (with $K = 30$). Comment on what these tell you about whether the errors are independent in this model.

(c) Diagnose the fit of the AR(1) model by using the overfitting strategy.

(d) Repeat part (b), but with the residuals from a (mean-centered) AR(2) model for the robot data. Comment on whether your conclusions are any different.