A stochastic process is a collection of random variables indexed by time.

We begin with a review of probability.

**Chapter 1: Review of Probability Theory**

**Defn:** An experiment is a random phenomenon whose outcome is not predictable in advance.

**Defn:** The sample space (denoted $\mathcal{S}$) of an experiment is the set of all possible outcomes.

**Example 1:** Consider the experiment of rolling 2 dice. The sample space consists of pairs $(i,j)$ such that
Example 2: Consider the experiment of measuring the lifetime of a part. The sample space is:

**Defn.** Any subset $E$ of the sample space $S$ is called an event.

Example 1: Let $E$ be the event that the sum of the 2 dice is prime. Then $E = \ldots$

Example 2: Let $E$ be the event that the part lasts between 2 and 4 years. Then $E = \ldots$

**Defn.** The union of two events $E$ and $F$ is denoted $E \cup F$ and is the set of outcomes that belong to $E$ or to $F$ (or to both).
Defn. The **intersection** of two events $E$ and $F$ is denoted and is the set of outcomes that belong to both $E$ and $F$.

**Example 1:** Let $F$ be the event that the sum of the 2 dice is even. Then $EF = \emptyset$.

**Note:** If no outcomes belong to both $E$ and $F$, we say that $EF = \emptyset$ (the **null event**) and that $E$ and $F$ are **mutually exclusive**.

We can consider unions and intersections of a sequence of events $\{E_n\}$, $n=1, 2, \ldots$:

$$\bigcup_{n=1}^{\infty} E_n$$

is

$$\bigcap_{n=1}^{\infty} E_n$$

is
Defn. The complement of $E$, denoted $E^c$, is the set of outcomes in $S$ that do not belong to $E$.

Note: $S^c = \ldots$

1.3 Probabilities of Events

- The probability of event $E$, denoted $P(E)$, satisfies the conditions:
  
  (i) 
  (ii) 
  (iii) For any sequence of events $E_1, E_2, \ldots$ such that $E_n E_m = \emptyset$ for $n \neq m$, then:

- When outcomes are equally likely, probabilities can often be readily calculated in simple problems:

Example 1: $P(E) =$
Basic Probability Rules

Complement Rule: \( P(E^c) = \)

Additive Rule:

Extended Additive Rule:

Inclusion-Exclusion Identity:

1.4 Conditional Probabilities

- The conditional probability of \( E \) given \( F \) is denoted \( P(E|F) \) and is the probability that our outcome is in \( E \), given the fact that our outcome is known to be in \( F \).
Conditional Probability Formula:

Example 1: It can be shown that 
\[ P(F) = \frac{1}{2} \] (verify). Then the probability that the sum is prime, given that it is even, is:

The probability the sum is even, given that it is prime, is:

Multiplicative Rule:
1.5 Independent Events

Defn: Two Events E and F are independent if and only if

Note: (*) is equivalent to:

and also equivalent to:

- So E and F are independent if and only if (*) , (**), or (***) are true (they will all be true or will all be false).

Example 1: Are E and F independent?
Independence of More than Two Events

- Events $E_1, E_2, \ldots, E_n$ are (jointly) independent if for any subset $E_{1}', E_{2}', \ldots, E_{r}'$, $r \leq n$:

Example 3: Consider an urn with four balls labeled 1, 2, 3, and 4. We draw a ball at random. Define the events:

$E = \{1, 2\}$  $F = \{1, 3\}$  $G = \{1, 4\}$

Then

So $E, F, G$ are

But

So $E, F,$ and $G$ are
1.6 Bayes' Formula

- If $E$ and $F$ are two events, note

More Than Two Events:
- Suppose $F_1, F_2, \ldots, F_N$ are mutually exclusive events such that $\bigcup_{i=1}^{n} F_i = S$. Then
Example (#39): Stores A, B, C have 50, 75, and 100 employees and, respectively, 50%, 60%, and 70% of these are women. One employee resigns and she is a woman. What is the probability she works in store C? Assume resignations are equally likely among all employees, regardless of sex.

Let $E =$

$F_1 =$

$F_2 =$

$F_3 =$
\[ P(\quad) = \quad \]

Note