Chapter 2: Random Variables

Defn. A random variable (r.v.) is a function that maps the set of outcomes in the sample space to a set of real numbers.

Example 1 (two dice): Let the r.v. $X$ be the sum of two fair dice. Then $X$ is the function:

<table>
<thead>
<tr>
<th>Outcome</th>
<th>$X$</th>
</tr>
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</table>

Defn. A discrete r.v. is one that takes on a finite or countably infinite number of values.
Defn: A **continuous r.v.** is one that takes on a continuum of values.

**Example 2:** If the r.v. $X$ measures the lifetime of a part, then $X$ is

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Defn: The **cumulative distribution function (cdf)** of a r.v. $X$ is defined as:

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**Note:** $F(b)$ is a nondecreasing function of $b$.

$$
\lim_{b \to -\infty} F(b) = \quad \text{and} \quad \lim_{b \to \infty} F(b) = 
$$

**Also:** $P(a < X \leq b) =$

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2.2 **Discrete Random Variables**

Defn. The **probability mass function (pmf)** of a discrete r.v. $X$ is
Note \( p(a) \) is positive for a finite or countable number of values \( a \). If \( X \) can take on values \( x_1, x_2, \ldots \), then \( \sum_{i=1}^{\infty} p(x_i) = \)

- The cdf of a discrete r.v. \( X \) is
  \[ F(a) = \sum_{\text{all } x_i \leq a} p(x_i) \]

2.3 Continuous Random Variables

- The probability density function (pdf) of a continuous r.v. \( X \) is a nonnegative function \( f(x) \) such that

  for any subset \( B \) of the real line.

  Note \( P(a \leq X \leq b) = \)

  and \( \int_{-\infty}^{\infty} f(x) \, dx = \)
The cdf of a continuous r.v. $X$ is and

Note that for a small $\epsilon > 0$, 

2.4 Expectation of a Random Variable

- If $X$ is a discrete r.v. with pmf $p(x)$, then the expected value of $X$ is

- If $X$ is a continuous r.v. with pdf $f(x)$, then the expected value of $X$ is
Expectation of a Function of a r.v.

- If \( g(X) \) is any function of a r.v. \( X \), then

\[
E[aX + b] = \]

for any constants \( a \) and \( b \).

- The variance of \( X \) is defined as

2.6  **Moment Generating Functions**

**Defn.** The moment generating function (mgf) of a r.v. \( X \) is denoted \( \phi(t) \) and defined as:
Note: The m-th derivative of $\phi(t)$, evaluated at $t=0$, equals $E[X^m]$, which is called the m-th moment of $X$.

So

The mgf of a r.v. uniquely determines its distribution.
- These common distributions are all familiar from STAT 511.
- Note the parameterization of the exponential and gamma here is different than in STAT 511:
  - Here, the rate parameter $\lambda$ is the same as $\frac{1}{\beta}$ where $\beta$ was the scale parameter in the gamma.
  - The shape parameter of the gamma is here denoted $n$ (we used $\alpha$ in STAT 511).

2.5 Jointly Distributed r.v.'s

Sometimes we are interested in probabilities involving more than one r.v.

Defn: If $X$ and $Y$ are r.v.'s, the joint cdf of $X$ and $Y$ is