- Find the expected number of families checked in at the beginning of day n if there are initially i families checked in.
What are the stationary probabilities $\pi_j$ of $\{X_n\}$?
Limiting Probabilities

Recall Example 1: A two-state Markov chain with transition probability matrix

\[ P = \begin{bmatrix} 0.5 & 0.5 \\ 0.3 & 0.7 \end{bmatrix} \]

Note

\[ P^4 = \]

\[ P^8 = \]

\[ P^{12} = \]
It appears $P_{ij}^n$ converges to some fixed value as $n \to \infty$, not depending on the initial state $i$.

Such limiting probabilities do not always exist.

**Example:** Consider a two-state Markov chain with transition probability matrix

$$P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

This chain continually

But $P_{00}^n =$

So $P_{00}^n$

**Defn.** An irreducible chain is periodic if it can only return to a state in a multiple of $d > 1$ steps.

In the previous example, the period $d =$
An irreducible chain that is not periodic is called aperiodic.

Theorem: For any aperiodic Markov chain, the limiting probabilities:
(i) exist;
(ii) do not depend on the initial state;
(iii) equal the long-run proportions, i.e.,

Proof: (part (iii)):
Application: (The Gambler’s Ruin Problem)
- This is related to the Random Walk process discussed earlier.
- Suppose a gambler makes successive and independent bets, each time having probability \( p \) of winning $1 and probability \( q = 1 - p \) of losing $1. Let \( X_n \) = the gambler’s fortune at time \( n \). The gambler’s goal is to amass \( N \) dollars; what is the probability he will reach his goal before going broke?
- The classes are:
  and the transition probabilities are
- The absorbing classes are _______ and the other class is _______.
- Each _______ state is visited only finitely often, so eventually:

- Let $P_i$ be the probability that, if the gambler begins with $i$ dollars, he eventually reaches his goal of $N$.
- In Section 4.5.1, it is shown that:
Example: A roulette player has chance $\frac{18}{38}$ of winning each bet. He enters the casino with $60$ and will play successive $10$-dollar bets until his fortune reaches $100$ or he goes broke. What is the probability that he goes broke?