Related Result: Given that $S_n = t$, the set $S_1, \ldots, S_{n-1}$ (taken as unordered) has the distribution of a sample of

- This can be proved similarly to the previous theorem.

5.4 Generalizations of the Poisson Process

- Instead of a constant rate λ, we could allow the rate to vary with t:

Defn: A counting process $\{N(t)\}$ is a nonhomogeneous (or nonstationary) Poisson Process with intensity function $\lambda(t)$, $t \geq 0$, if:

(i)

(ii)

(iii)

(iv)

The function

is called the mean value function of the process.

- If $\{N(t)\}$ is a nonhomogeneous Poisson process, then $N(t+s) - N(s)$ is a Poisson r.v. with mean $m(t+s) - m(s)$.

<u>Example:</u> A hot dog stand's customer arrivals follow a nonhomogeneous Poisson process with

$$\lambda(t) = \begin{cases} 5 + 5t, & 0 \le t \le 3 \\ 20, & 3 < t < 5 \\ 20 - 2(t-5), & 5 \le t \le 9 \end{cases}$$

where $t = \#$ hours past 8:00 a.m.

Picture:

- What is the probability of exactly 21 customers between 9:00 and 11:00 a.m.?

- What is the expected number of customers between 9:00 and 11:00 a.m.?

- <u>Defn</u>: A stochastic process $\{X(t)\}$ is a <u>compound Poisson process</u> if

- It is straightforward to show by conditioning that:

<u>Example 1 again:</u> Recall the customer entrance process was a Poisson process with rate $\lambda = 25$ per hour. Suppose the amount Y_i each customer spends is gamma $(300, 10)$. Find the expected value and variance of the total amount spent in the store in 8 hours.

<u>Defn</u>: A counting process $\{N(t)\}$ is a <u>conditional</u> (or <u>mixed</u>) <u>Poisson</u> <u>process</u> if there exists a positive r.v. L such that, conditional on $L = \lambda$, $\{N(t)\}$ is a Poisson process.

Example: If an insurance company believes its policyholders have some rating L taking on some value λ, then a policyholder with rating λ will make claims according to a Poisson process with rate λ. Then the number of claims by a random customer in t years, $t \geq 0$, is a conditional Poisson process.

- If L is continuous with pdf $g(\lambda)$:

- The conditional pdf of L given that $N(t) = n$ is:

- A conditional Poisson process has stationary increments, but not independent increments. Why?