Related Result: Given that $S_n = t$, the set $S_1, \ldots, S_{n-1}$ (taken as unordered) has the distribution of a sample of

- This can be proved similarly to the previous theorem.

5.4 Generalizations of the Poisson Process

- Instead of a constant rate $\lambda$, we could allow the rate to vary with $t$.

Defn: A counting process $\{N(t)\}$ is a nonhomogeneous (or nonstationary) Poisson Process with intensity function $\lambda(t)$, $t \geq 0$, if:

(i)
(ii)
(iii)
(iv)
The function

is called the mean value function of the process.

- If \( \{N(t)\} \) is a nonhomogeneous Poisson process, then \( N(t+s) - N(s) \) is a Poisson r.v. with mean \( m(t+s) - m(s) \).

**Example:** A hot dog stand's customer arrivals follow a nonhomogeneous Poisson process with

\[
\lambda(t) = \begin{cases} 
5 + 5t, & 0 \leq t \leq 3 \\
20, & 3 < t < 5 \\
20 - 2(t-5), & 5 \leq t \leq 9 
\end{cases}
\]

where \( t = \) #hours past 8:00 a.m.

Picture:
- What is the probability of exactly 21 customers between 9:00 and 11:00 a.m.?

- What is the expected number of customers between 9:00 and 11:00 a.m.?

- **Defn:** A stochastic process \( \{X(t)\} \) is a **compound Poisson process** if
It is straightforward to show by conditioning that:

Example 1 again: Recall the customer entrance process was a Poisson process with rate $\lambda = 25$ per hour. Suppose the amount $Y_i$ each customer spends is gamma $(300, 10)$. Find the expected value and variance of the total amount spent in the store in 8 hours.

**Defn:** A counting process $\{N(t)\}$ is a conditional (or mixed) Poisson process if there exists a positive r.v. $L$ such that, conditional on $L=\lambda$, $\{N(t)\}$ is a Poisson process.
Example: If an insurance company believes its policyholders have some rating $L$ taking on some value $\lambda$, then a policyholder with rating $\lambda$ will make claims according to a Poisson process with rate $\lambda$. Then the number of claims by a random customer in $t$ years, $t \geq 0$, is a conditional Poisson process.

- If $L$ is continuous with pdf $g(\lambda)$:

- The conditional pdf of $L$ given that $N(t) = n$ is:

- A conditional Poisson process has stationary increments, but not independent increments. Why?