Example #6.4 (Linear Growth Model with Immigration)

- This is a common model for population growth.
- Each individual has exponential birth rate λ, so $\lambda_n = n\lambda$.
- There is an exponential rate of increase of θ from immigration.
- Each individual has exponential death rate μ, so $\mu_n = n\mu$.
- We let $X(t)$ denote the population size at time t, and suppose $X(0) = i$.
- We now find $M(t) = E[X(t)]$.

$M(t+h) =$

- Note for h small,

So

So since $M(t) = E[X(t)]$,

To obtain the constant K, we use the fact that

Example #6.5 (M/M/1 Queueing Model):

- Suppose customers arrive at a single-server station according to a Poisson process with rate λ. If the server is available, the customer is served immediately. Otherwise, he waits in the queue (line). Customers in the queue are served in order of their arrival. The service times are exponential with rate μ.
- Let $X(t)$ denote the number of customers in the system at time t.

- Then $X(t)$ is

- This is an M/M/1 queueing system:

First:

Second:

Third:

- Consider an M/M/s queueing system (same situation, but with s servers):

- Other queueing systems are $M/G/1$, $G/M/1$, etc. The "G" denotes a ________ service or interarrival distribution.

Times Until Transitions

- Consider a birth and death process with rates $\{\lambda_n\}$ and $\{\mu_n\}$. Let T_i $(i \geq 0)$ denote the time (starting in state i) that it takes to reach state $i+1$.
- We now compute $E[T_i]$.

$E[T_0] =$ since $T_0 \sim$

For $i > 0$, condition on the result of the first transition:

Let $W_i =$

- Note the time until the first transition is distributed as

So

Example (M/M/1): Consider the M/M/1 queueing model with $\lambda = 4.0$ and $\mu = 2.5$ per hour. Find the expected time to go from 2 customers in the system to 5 customers in the system.

- Note this is:

We know