6.9 Computing Transition Probabilities

- Section 6.9 discusses a method for approximating $P_{ij}(t)$ for any states i, j and any $t \geq 0$.

- Let $r_{ij} =$

and let the matrix R have the $\{r_{ij}\}$ in its (i, j) positions.

- Also, let $P(t)$ contain $P_{ij}(t)$ in its (i, j) position.

- See Section 6.9 for the details about these two methods of approximating the matrix $P(t)$.

<u>Method 1</u>: If n is large,

Method 2: If n is large,

Example 1 again (Two-window restaurant):

- If the system is empty at 7:00, what is the probability that it will be empty at 7:05? What is the probability there will be a customer at window 1 at 7:05?

6.5 Limiting Probabilities

- Consider the probability that a Markov chain will be in state j at time t, where time t is far into the future.
- Often this long-run probability does not depend on the initial state that the chain is in.

Defn: If it exists, the limiting probability P_j is

where P_j is independent of initial state i.

- To find P_j, we use the Forward Equations

- If we can interchange limit and summation,

- Now,

- We can solve these equations for the $\{P_j\}$.
- A sufficient condition for the limiting probabilities to exist is for both:

- If this condition holds, the P_j's exist and the chain is called ________.

<u>Note</u>: P_j is the long-run proportion of time that the chain is in state j.

<u>Note</u>: is the overall rate at which the process leaves state j, and is the overall rate at which the process enters state j.

- Since these quantities are set equal, the rates at which the process enters and leaves j are equal. Thus these equations are called the ________ ________.

- The P_j's are also called the ________ probabilities, since if the initial state is chosen according to the probabilities $\{P_j\}$, then for all t:

Limiting Probabilities for the Birth and Death Process

- For the birth and death process,

State j | Balance Equation

So

Example 6.4 again: Recall the Linear Growth model with immigration. For $n \geq 0$, we had:

So the limiting probabilities exist, and are found by the previous formula, if and only if

Using rules for convergence of series, this holds if and only if . So

Example 1 again (Two-window restaurant):

Recall we had

State

Balance Equation

Example (M/M/1 queue): Recall $\lambda_n = \lambda$ for $n \geq 0$ and $\mu_n = \mu$ for $n \geq 1$. So for $n \geq 1$,

- If customers in a M/M/1 system arrive at rate 4 per minute and depart at rate 5 per minute, how often are there exactly 3 people in the system?