STAT 521 HW 4 Example Solutions

(2) a) $\lambda = 2$. $P(T > 0.5) = e^{-2(0.5)} = e^{-1} = \boxed{0.368}$

b) $P(T \geq 12.5 \mid T > 12) = P(T > 0.5) = \boxed{0.368}$

(3) $E(T) = E(W_1 + W_2 + W_3 + W_4 + W_5 + S)$

$= 6\left(\frac{1}{\mu}\right) = \boxed{\frac{6}{\mu}}$

(4) a) 0, since C cannot be finished before A.

b) $P[$A still there after other two have left$]$

$= P[$A has service time 3, and B and C both have service time 1$]$

$= \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = \boxed{\frac{1}{27}}$

c) Let X_A, X_B, X_C be the service times for A, B, and C. $P[X_A > X_B] = \frac{1}{2}$. We note $P[X_A^* > X_C] = \frac{1}{2}$, where X_A^* is the remaining service time for A after B has left, and X_A^* is also exponential with mean $\frac{1}{\mu}$.

So the needed probability is $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \boxed{\frac{1}{4}}$.

(5) $P[$Machine 1 fails first$]$

$= P[\text{Machine 1 fails first} \mid T_1 < t]\, P[T_1 < t]$

$+ P[\text{Machine 1 fails first} \mid T_1 \geq t]\, P[T_1 \geq t]$.

(where T_1 = lifetime of Machine 1)

$$= (1)(1-e^{-\lambda_1 t}) + \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right)(e^{-\lambda_1 t})$$

$$= \boxed{1 - e^{-\lambda_1 t} + \frac{\lambda_1 e^{-\lambda_1 t}}{\lambda_1+\lambda_2}}$$

(8) a) Let S_1 = your service time at server 1
S_A = A's service time at server 2

$$P_A = P(S_1 < S_A) = \boxed{\frac{\mu_1}{\mu_1+\mu_2}}$$

b) Let S_B = B's service time at server 2

$P_B = 1 - P[$B is done when you move to server 2$]$

$= 1 - P[$A is done when you move$]\, P[$B done when you move | A done when you move$]$

$= 1 - P[S_A < S_1]\, P[S_B < S_1 \mid S_A < S_1]$

$$= 1 - \left(\frac{\mu_2}{\mu_1+\mu_2}\right)\left(\frac{\mu_2}{\mu_1+\mu_2}\right) = \boxed{1 - \left(\frac{\mu_2}{\mu_1+\mu_2}\right)^2}$$

(8) c) See next page.

(6) Let W = additional lifetime of the "other pet".

$$E(W) = E(W \mid \text{dog died})P(\text{dog died}) + E(W \mid \text{cat died})P(\text{cat died})$$

$$= \left(\frac{1}{\lambda_c}\right)\left(\frac{\lambda_d}{\lambda_d+\lambda_c}\right) + \left(\frac{1}{\lambda_d}\right)\left(\frac{\lambda_c}{\lambda_c+\lambda_d}\right)$$

(7) Let S_1 = service time for 1:00 patient
S_2 = service time for 1:30 patient
T = total time 1:30 patient spends in office

$$E(T) = E(T \mid S_1 \le 30)P(S_1 \le 30) + E(T \mid S_1 > 30)P(S_1 > 30)$$

$$= E(S_2)(1 - e^{-30/30}) + [E(S_1)+E(S_2)](e^{-30/30})$$

$$= (30)(.632) + [30+30](.368) = \boxed{41.04 \text{ minutes}}$$

8 c). Use the hint:

$$E(T) = E(S_1) + E(S_2) + E(W_A) + E(W_B)$$

$$= \frac{1}{\mu_1} + \frac{1}{\mu_2} + P_A\left(\frac{1}{\mu_2}\right) + P_B\left(\frac{1}{\mu_2}\right)$$

where P_A and P_B were found in parts (a) + (b).