1(a) \( P_{02} = 0.6 \)

(b) No. The classes are \( \{0, 2\} \) and \( \{1\} \). \( \{1\} \) is recurrent (since it is absorbing) and \( \{0, 2\} \) is transient, since the chain might leave this and enter the absorbing state.

(c) \( 1 - P_{00}^4 = 1 - 0.0535 = 0.9465 \)

(d) \( P[X_4 = 2] = P_{02} \alpha_0 + P_{12} \alpha_1 + P_{22} \alpha_2 \)
\( = (0.41158)(0.3) + (0)(0.3) + (0.5386)(0.4) = 0.3402 \)

2(a) \( P = \begin{bmatrix} 0.4 & 0.6 \\ 0.3 & 0.7 \end{bmatrix} \)

(b) Both states are in a single class; they communicate.
\( 1 \rightarrow 0 \) and \( 0 \rightarrow 1 \), so \( 0 \leftrightarrow 1 \).

(c) \( \pi_1 = 0.6 \pi_0 + 0.7 \pi_1 \) and \( \pi_0 + \pi_1 = 1 \)
\( \Rightarrow \pi_1 = 0.6(1 - \pi_1) + 0.7 \pi_1 \Rightarrow \pi_1 = 0.1 \pi_1 + 0.6 \)
\( \Rightarrow 0.9 \pi_1 = 0.6 \Rightarrow \pi_1 = \frac{2}{3} \)

(d) Positive recurrent, since this is an irreducible finite Markov chain (and all \( \pi_j > 0 \) for \( j = 0, 1 \)).

3(a) Let \( T = \) the number of accidents tomorrow.
\( E(T) = E(T \mid \text{rain}) P(\text{rain}) + E(T \mid \text{dry}) P(\text{dry}) \)
\( = (7)(0.6) + (3)(0.4) = 5.4 \)

(b) \( P(T=0) = P(T=0 \mid \text{rain}) P(\text{rain}) + P(T=0 \mid \text{dry}) P(\text{dry}) \)
\( = \left[ \frac{e^{-7} 7^0}{0!} \right](0.6) + \left[ \frac{e^{-3} 3^0}{0!} \right](0.4) = 0.6 e^{-7} + 0.4 e^{-3} \)
\( = 0.0205 \)
\[ \text{4) } P(D | \text{Pos}) = \frac{P(\text{Pos} | D) P(D)}{P(\text{Pos} | D) P(D) + P(\text{Pos} | D^c) P(D^c)} \]
\[ = \frac{(0.95)(0.02)}{(0.95)(0.02) + (0.10)(0.98)} = \frac{0.019}{0.117} = 0.162 \]

\[ \text{5) Let } i = 4 \text{ and let 1 unit} = \$100 \text{ in the Gambler's Ruin setup. Condition on his mood:} \]
\[ P(\text{goal}) = P(\text{goal} | \text{conf}) P(\text{conf}) + P(\text{goal} | \text{doubt}) P(\text{doubt}) \]
\[ = \left[ 1 - \left(\frac{0.4}{0.6}\right)^4 \right] (0.7) + \left(\frac{4}{7}\right) (0.3) \]
\[ = (0.8234)(0.7) + (0.5714)(0.3) = 0.748 \]

\[ \text{6a) } P_T = \begin{bmatrix} 0.1 & 0.2 & 0.2 \\ 0.1 & 0.6 & 0.1 \\ 0.4 & 0.2 & 0.1 \end{bmatrix} \]

b) \[ S_{2,1} = 0.533 \]

c) 0

d) \[ f_{2,1} = \frac{S_{21} - S_{21}}{S_{11}} = \frac{0.533 - 0}{1.393} = 0.3826 \]

e) \[ f_{22} = \frac{S_{22} - S_{22}}{S_{22}} = \frac{2.992 - 1}{2.992} = 0.6658 \]

EC) Britain + Russia