1. Let $Y_{11}, \ldots, Y_{1n_1}$ be a sample from a population with mean $\mu_1$ and variance $\sigma_1^2$. Let $Y_{21}, \ldots, Y_{2n_2}$ be a sample from another population with mean $\mu_2$ and variance $\sigma_2^2$. Define

$$
\bar{Y}_1 = \frac{1}{n_1} \sum_{j=1}^{n_1} Y_{1j}, \quad \bar{Y}_2 = \frac{1}{n_2} \sum_{j=1}^{n_2} Y_{2j}.
$$

(a) Find $E(\bar{Y}_1 - \bar{Y}_2)$.

(b) If $\bar{Y}_1$ and $\bar{Y}_2$ are independent, find $\text{var}(\bar{Y}_1 - \bar{Y}_2)$.

(c) If the two populations are normal (and $\bar{Y}_1$ and $\bar{Y}_2$ are independent), then does $\bar{Y}_1 - \bar{Y}_2$ have a normal distribution? Explain why or why not.

2. Suppose $Y_1, Y_2, \ldots, Y_n$ are independent random variables with mean $\mu$ and variance $\sigma^2$.

(a) Show that

$$(n - 1)S^2 = \sum_{i=1}^{n} Y_i^2 - n\bar{Y}^2.$$

(b) Show that $E(S^2) = \sigma^2$. (Hint: Use the fact that $\text{var}(Y) = E(Y^2) - [E(Y)]^2$.)

3. Let $Y_1, Y_2, Y_3$ be independent random variables with means $\mu_1, \mu_2, \mu_3$ and a common variance $\sigma^2$. Define

$$
\bar{Y} = \frac{1}{3} \sum_{i=1}^{3} Y_i.
$$

(a) Find the covariance between $Y_1 - \bar{Y}$ and $\bar{Y}$.

(b) Find the expected value of $(Y_1 + 2Y_2 - Y_3)^2$.

4. Let $Y_1$ and $Y_2$ be random variables with expected values $\mu_1$ and $\mu_2$ and variances $\sigma_1^2$ and $\sigma_2^2$.

(a) Show that $\text{cov}(Y_1 + Y_2, Y_1 - Y_2) = \sigma_1^2 - \sigma_2^2$.

(b) If $W = Y_1 + Y_2$ and $V = Y_1 - Y_2$, then under what condition(s) can we be assured that $W$ and $V$ are independent random variables?