13.11. The uniform distribution is shown in Figure 13.12 (Moore and Notz, pp 310). This is a population density curve, but it is constant. The height is 0.2 and the length is 5.
(a) The total area under the curve is

$$5 \times 0.2 = 1.$$

We remember from geometry that the area of a rectangle is the base (5) times the height (0.2).
(b) The mean is the balance point. This curve would balance at 2.5, exactly in the middle. The median is the equal-areas point, splitting the area under the curve into equal 50% regions. The median is also 2.5. We know the mean and median are equal because the population density curve is symmetric.
(c) 20%. This is the area under the curve over that region:

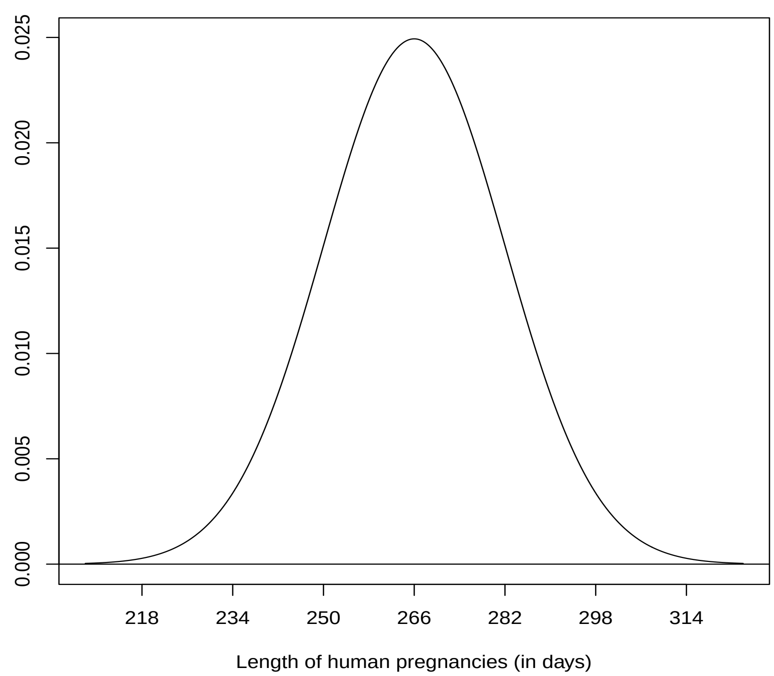
$$1 \times 0.2 = 0.2.$$

The area is 0.2 as a proportion. As a percentage, this is 20%.
(d) 30%. This is the area under the curve over that region:

$$1.5 \times 0.2 = 0.3.$$

The area is 0.3 as a proportion. As a percentage, this is 30%.

13.15. Here is the population density curve for the length of human pregnancies:



This is a normal distribution with mean $\mu = 266$ days and standard deviation $\sigma = 16$ days. In the graph above, tick marks are shown ± 1 standard deviation, ± 2 standard deviations, and ± 3 standard deviations from the mean.

(a) From the 68-95-99.7 Rule, we know that 99.7% of the pregnancy lengths will be within 3 standard deviations of the mean. We calculate:

$$\mu + 3\sigma = 266 + 3(16) = 314$$

and

$$\mu - 3\sigma = 266 - 3(16) = 218.$$

Therefore, **99.7% of human pregnancy lengths will be between 218 and 314 days.**

(b) From the 68-95-99.7 Rule, we know that 95% of the pregnancy lengths will be within 2 standard deviations of the mean. We calculate:

$$\mu + 2\sigma = 266 + 2(16) = 298$$

and

$$\mu - 2\sigma = 266 - 2(16) = 234.$$

Therefore, 95% of human pregnancy lengths will be between 234 and 298 days. This means

- 2.5% of the human pregnancy lengths will be more than 298 days.
- 2.5% of the human pregnancy lengths will be less than 234 days.

Therefore, **the longest 2.5% of all pregnancies will be more than 298 days.**

(c) From the 68-95-99.7 Rule, we know that 68% of the pregnancy lengths will be within 1 standard deviation of the mean. We calculate:

$$\mu + \sigma = 266 + 16 = 282$$

and

$$\mu - \sigma = 266 - 16 = 250.$$

Therefore, 68% of human pregnancy lengths will be between 250 and 282 days. This means

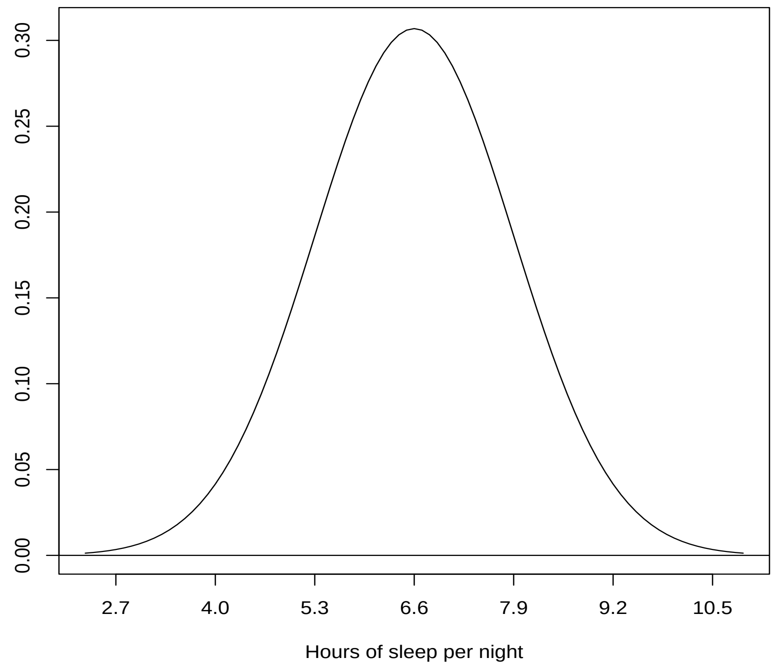
- 16% of the human pregnancy lengths will be more than 282 days.
- 16% of the human pregnancy lengths will be less than 250 days.

Therefore, **the shortest 16% of all pregnancies will be less than 250 days.**

Note: I used the R code below to make the normal distribution on the last page:

```
x = seq(208,324,0.1)
pdf = dnorm(x,266,16)
plot(x,pdf,type="l",xlab="Length of human pregnancies (in days)",ylab="",
    xaxp=c(218,314,6))
abline(h=0)
```

13.24. Here is the population density curve for the number of hours of sleep per school night:



This is a normal distribution with mean $\mu = 6.6$ hours and standard deviation $\sigma = 1.3$ hours. In the graph above, tick marks are shown ± 1 standard deviation, ± 2 standard deviations, and ± 3 standard deviations from the mean.

(a) From the 68-95-99.7 Rule, we know that 68% of the students will be within 1 standard deviation of the mean. We calculate:

$$\mu + \sigma = 6.6 + 1.3 = 7.9$$

and

$$\mu - \sigma = 6.6 - 1.3 = 5.3.$$

Therefore, 68% of the students will sleep between 5.3 and 7.9 hours. This means

- 16% of the students will sleep less than 5.3 hours.
- 16% of the students will sleep more than 7.9 hours.

Therefore, **16% of the students will sleep more than 7.9 hours.**

From the 68-95-99.7 Rule, we know that 95% of the students will be within 2 standard deviations of the mean. We calculate:

$$\mu + 2\sigma = 6.6 + 2(1.3) = 9.2$$

and

$$\mu - 2\sigma = 6.6 - 2(1.3) = 4.0.$$

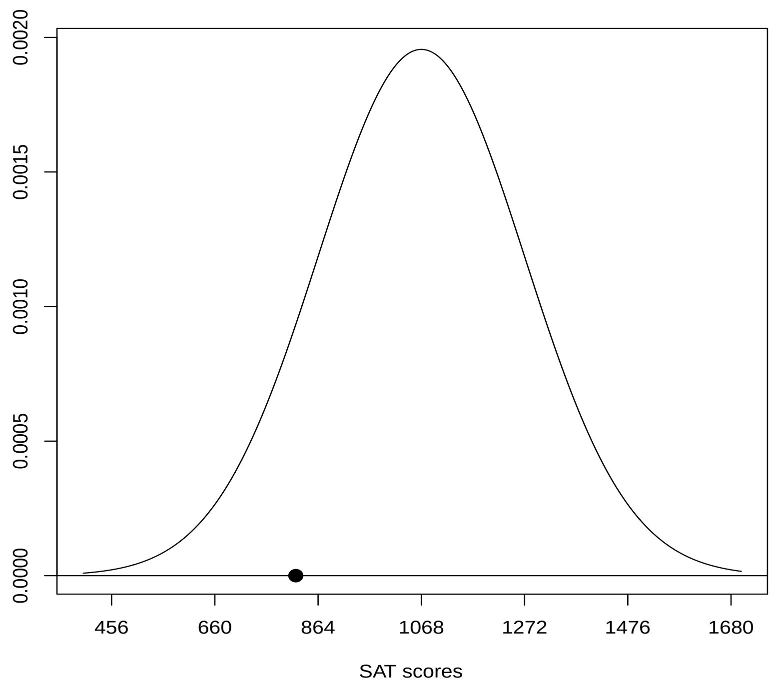
Therefore, 95% of the students will sleep between 4.0 and 9.2 hours. This means

- 2.5% of the students will sleep less than 4.0 hours.
- 2.5% of the students will sleep more than 9.2 hours.

Therefore, **2.5% of the students will sleep less than 4.0 hours.**

(b) 68% of the students will sleep between 5.3 and 7.9 hours. We calculated this above.

13.26. Here is the population density curve for SAT scores for college-bound seniors in 2018:

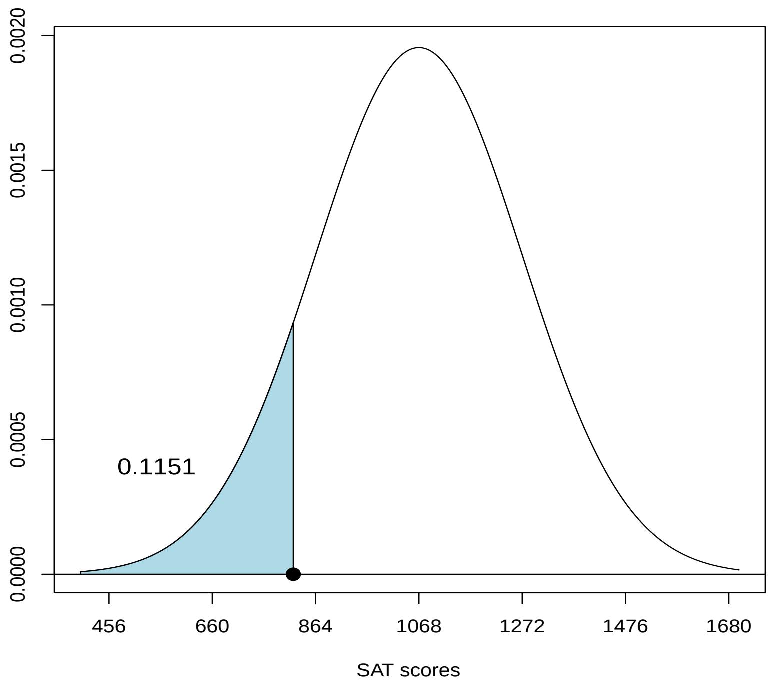


This is a normal distribution with mean $\mu = 1068$ and standard deviation $\sigma = 204$. In the graph above, tick marks are shown ± 1 standard deviation, ± 2 standard deviations, and ± 3 standard deviations from the mean. A solid circle at 820 is shown.

We want to find the **area to the left** of 820−this area corresponds to the percentage of college-bound seniors in the population who scored less than 820.

1. Calculate the standard score:
$$z = \frac{\text{observation} - \text{mean}}{\text{standard deviation}} = \frac{820 - 1068}{204} \approx -1.2.$$
2. Look $z = -1.2$ up on Table B and read off the percentage:
$$z = -1.2 \implies \text{Percentage} = 11.51\%.$$

Therefore, 11.51% of all college-bound seniors scored less than 820 on the SAT in 2018. See the graph on the next page.



Implementation in R:

```
> pnorm(-1.2)
[1] 0.1151
```

13.32. The population density curve for the S&P500 annual returns (in percentages) since 1945 is shown on the top of the next page (left). This is a normal distribution with mean $\mu = 12.5\%$ and standard deviation $\sigma = 17.8\%$. In the graph above, tick marks are shown ± 1 standard deviation, ± 2 standard deviations, and ± 3 standard deviations from the mean.

(a) From the 68-95-99.7 Rule, we know that 95% of the annual returns are within 2 standard deviations of the mean. We calculate:

$$\mu + 2\sigma = 12.5 + 2(17.8) = 48.1$$

and

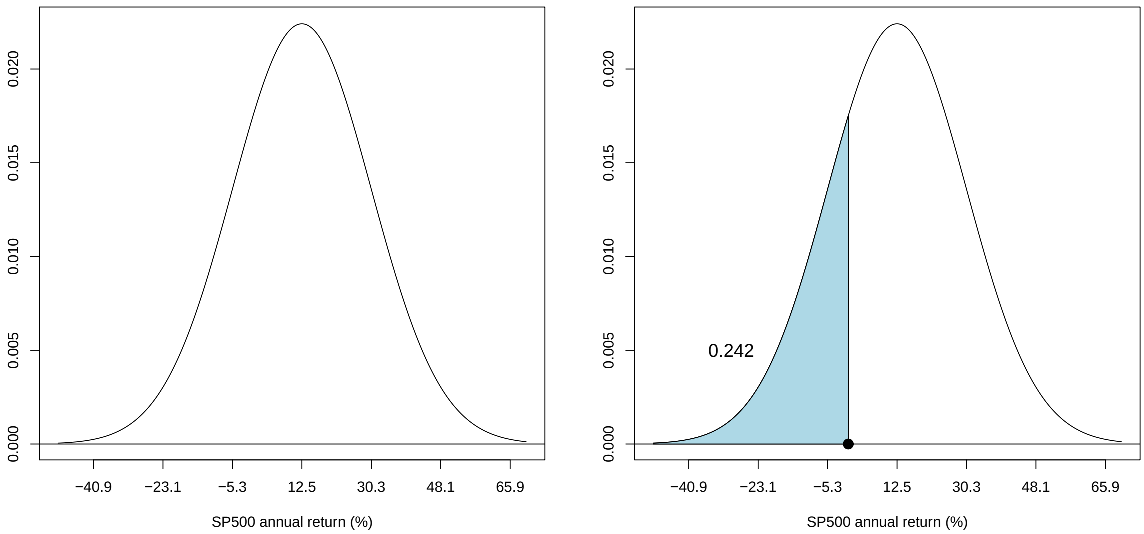
$$\mu - 2\sigma = 12.5 - 2(17.8) = -23.1.$$

Therefore, 95% of the S&P500 annual returns will be between -23.1% and 48.1%.

(b) We want to find the **area to the left** of 0%−this area corresponds to the percentage of years the S&P500 annual return will be negative (i.e., less than 0%).

1. Calculate the standard score:
$$z = \frac{\text{observation} - \text{mean}}{\text{standard deviation}} = \frac{0 - 12.5}{17.8} \approx -0.7.$$
2. Look $z = -0.7$ up on Table B and read off the percentage:
$$z = -0.7 \implies \text{Percentage} = 24.20\%.$$

Therefore, 24.20% of annual returns will be negative (i.e., 24.20% of the years will be "down years" where the market is down). See the graph on the next page (right).



Implementation in R:

```
> pnorm(-0.7)
[1] 0.2420
```

(c) We want to find the **area to the right** of 25%−this area corresponds to the percentage of years the S&P500 annual return will be 25% or more.

1. Calculate the standard score:

$$z = \frac{\text{observation} - \text{mean}}{\text{standard deviation}} = \frac{25 - 12.5}{17.8} \approx 0.7.$$

2. Look $z = 0.7$ up on Table B and read off the percentage:

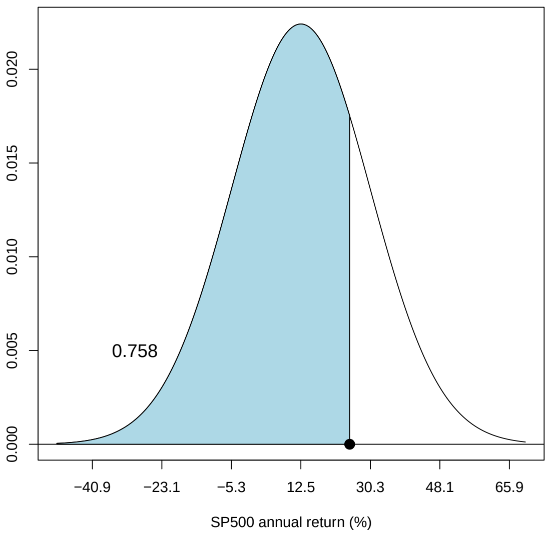
$$z = 0.7 \implies \text{Percentage} = 75.80\%.$$

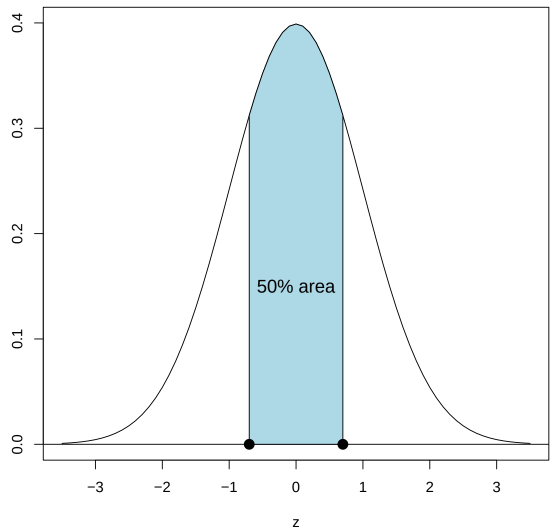
- This is the percentage of years the S&P500 annual return will be less than 25% (see next page, left).
- Therefore, the percentage of years the S&P500 annual return will be more than 25% is

$$100\% - 75.80\% = 24.20\%.$$

In R (which reports decimals; not percentages),

```
> 1-pnorm(0.7)
[1] 0.2420
```





13.33. Table B shows the standard normal distribution; i.e., a normal distribution with mean 0 and standard deviation 1. From Table B,

$$z = -0.7 \implies \text{Percentage} = 24.20\%$$
$$z = -0.6 \implies \text{Percentage} = 27.42\%.$$

Therefore, the 25th percentile (Q_1) is somewhere between $z = -0.7$ and $z = 0.6$ on the standard normal distribution. This means the first quartile Q_1 is approximately -0.6 to -0.7 standard deviations **below** the mean.

Because the standard normal distribution is symmetric, this means the 75th percentile (Q_3) is somewhere between $z = 0.6$ and $z = 0.7$. This means the third quartile Q_3 is approximately 0.6 to 0.7 standard deviations **above** the mean.

The graph above (right) shows the standard normal distribution with the quartiles identified.