Question 1.
(a) The sampling distribution of $\overline{Y}$ is

$$\overline{Y} \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right).$$

Here, the population mean $\mu = 130$, the population variance $\sigma^2 = 2.25$, and the sample size $n = 20$. Therefore,

$$\overline{Y} \sim \mathcal{N}\left(130, \frac{2.25}{20}\right).$$

(b) The standard error of $\overline{Y}$ is the (positive) square root of its variance; i.e.,

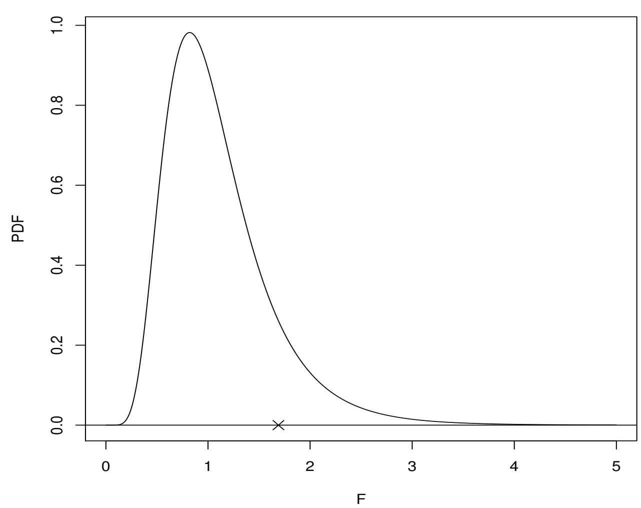
$$\text{se}(\overline{Y}) = \sqrt{\text{var}(\overline{Y})} = \sqrt{\frac{2.25}{20}} \approx 0.34.$$

(c) $t(19)$, a t distribution with 19 degrees of freedom
(d) $E(\overline{Y}) = 130$ and $E(S^2) = 2.25$. These statements are true because the sample mean $\overline{Y}$ is an unbiased estimator of the population mean μ (here, $\mu = 130$). Similarly, the sample variance S^2 is an unbiased estimator of the population variance σ^2 (here, $\sigma^2 = 2.25$).

Question 2.
(a) The ratio of the sample variances is

$$\frac{S_1^2}{S_2^2} = \frac{46.93}{27.81} \approx 1.69.$$

Here is the $F(19, 22)$ sampling distribution with the value $F = 1.69$ identified (by an $\times$):

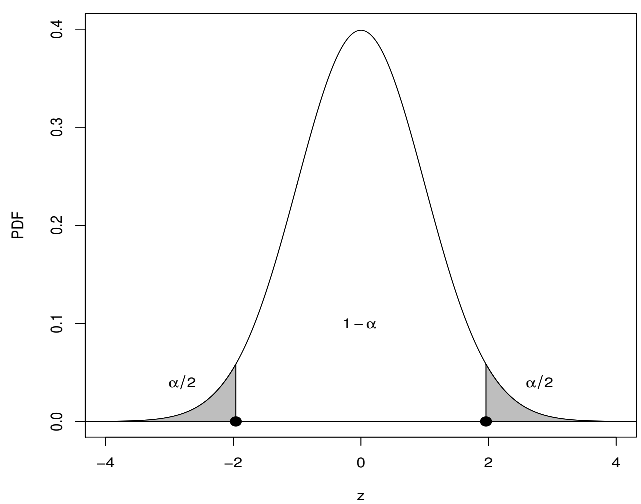


The plausibility of the equal population variance assumption can be assessed by characterizing how "reasonable" or "unreasonable" the value $F = 1.69$ is when compared to its sampling distribution, $F(19, 22)$. This value is certainly not unreasonable (i.e., it is not pinned down in the lower tail or far out in the upper tail). Therefore, I would say that there is little or no evidence against the equal population variance assumption.

(b) "Not robust to departures from normality" means that the sampling distribution result depends critically on the normality assumption (here, for each sample). In other words, if normality does not hold, then the sampling distribution result $F \sim F(19, 22)$ will not be accurate. We could check the normality assumption empirically by constructing normal qq plots for each sample of weight gains.
(c) Regardless of which interval you choose, the general conclusion is the same. There is evidence that the population mean weight gain for rats on the low dose of toxaphene μ_1 is larger than the population mean for rats receiving no toxaphene μ_2 (because the confidence intervals for $\mu_1 - \mu_2$ do not include "0" and include only positive values). For specificity (assuming the equal-variance interval),

- We are 95 percent confident that the population mean weight gain for rats on the low dose of toxaphene is between 20.48 and 27.96 grams larger than the population mean weight gain for rats not receiving toxaphene.

Question 3.
(a) A reasonable answer to the "population" question is "all legally-registered automobiles in CT." The sample is the 200 automobiles tested.
(b) This picture is taken directly from the notes:



The upper dark circle is $z_{\alpha/2}$. The lower dark circle is $-z_{\alpha/2}$.
(c) We can set

$$0.02 = 1.96\sqrt{\frac{\frac{124}{200}\left(1 - \frac{124}{200}\right)}{n}}$$

and solve for n. Note that I have used the sample proportion

$$\widehat{p} = \frac{124}{200}$$

in this calculation. You could alternatively substitute in 0.5 for this proportion (if you wanted to be conservative; i.e., maximize the sample size needed). After the algebra, we get

$$n = \left(\frac{1.96}{0.02}\right)^2 \frac{124}{200}\left(1 - \frac{124}{200}\right) \approx 2262.7.$$

She would need to sample 2263 automobiles from those legally-registered in CT.

Question 4.
(a) This is a matched pairs study because two measurements are obtained on each car:

- km/l measurement using radial tires
- km/l measurement using belted tires.

The fact that two measurements are made on the same car makes the samples dependent.
(b) The notation `diff` represents the data differences. For example, the first data difference is $4.2 - 4.1 = 0.1$. The second data difference is $4.7 - 4.9 = -0.2$, and so on. There are 12 data differences.

The 95 and 99 percent confidence intervals are interpreted as follows:

- When comparing radial to belted tires, we are 95 percent confident that the population mean difference in gasoline consumption, say $\mu_R - \mu_B$, is between 0.016 and 0.267 km/l.
- When comparing radial to belted tires, we are 99 percent confident that the population mean difference in gasoline consumption, say $\mu_R - \mu_B$, is between -0.035 and 0.318 km/l.

It should not bother you. The 99 percent confidence interval has more confidence! Therefore, it will be lengthier than the analogous 95 percent confidence interval. Interestingly, we do have evidence that the population means are different at the 95 percent confidence level (interval includes only positive values). However, at the 99 percent confidence level, we cannot assert that the population means are different.

Question 5.
(a) Let μ_1, μ_2, and μ_3 denote the population mean compression strengths of the respective box types (Type I, Type II, and Type III). The F statistic is used to test

$$H_0 : \mu_1 = \mu_2 = \mu_3$$
versus
$$H_1 : \text{the population means } \mu_i \text{ are not all equal.}$$

The very small p-value here (p-value = 0.000002509) provides very strong evidence to reject H_0. In other words, there is very strong evidence that the population means are not equal.

(b) The results show that (with overall 95 percent confidence)

- There is no evidence that the Type I population mean and Type II population mean are different. Why?
  - pairwise CI includes "0"
  - large (adjusted) p-value
- There is strong evidence that the Type III population mean is smaller than the Type I population and Type II population mean. Why?

  - pairwise CIs exclude “0” and contain only negative values
  - small (adjusted) p-values

If the goal was to maximize population mean compression strength, either Type I and Type II should be used. These types produce higher population means than Type III (with 95 percent confidence), but they themselves are not different from each other in terms of population mean compression strength.