Question 1.
Each flashlight gets two batteries. Define

$$\begin{aligned} A_1 &= \{\text{battery 1 has acceptable voltage}\} \\ A_2 &= \{\text{battery 2 has acceptable voltage}\}. \end{aligned}$$

We have $P(A_1) = P(A_2) = 0.9$.

(a) A flashlight is operational when **both** batteries have acceptable voltages. Therefore, the probability that a flashlight is operational is

$$P(A_1 \cap A_2) \overset{\text{indep}}{=} P(A_1)P(A_2) = 0.9(0.9) = 0.81.$$

(b) Think of each flashlight as a Bernoulli trial; i.e., independent operational statuses, same probability of being operational ($p = 0.81$), and operational/not. Define

$$Y = \text{ number of operational flashlights (out of 10).}$$

Then $Y \sim b(n = 10, p = 0.81)$ and

$$\begin{aligned} P(Y \geq 9) &= P(Y = 9) + P(Y = 10) \\ &= \binom{10}{9}(0.81)^9(0.19)^1 + \binom{10}{10}(0.81)^{10}(0.19)^0 \\ &= 0.2852 + 0.1216 = 0.4068. \end{aligned}$$

(c) Now, a flashlight is operational when **at least one** of the batteries has an acceptable voltage. Therefore, the probability that a flashlight is operational is

$$\begin{aligned} P(A_1 \cup A_2) &= P(A_1) + P(A_2) - P(A_1 \cap A_2) \\ &= 0.9 + 0.9 - 0.81 = 0.99 \end{aligned}$$

and

$$\begin{aligned} P(Y \geq 9) &= P(Y = 9) + P(Y = 10) \\ &= \binom{10}{9}(0.99)^9(0.01)^1 + \binom{10}{10}(0.99)^{10}(0.01)^0 \\ &= 0.0914 + 0.9044 = 0.9958. \end{aligned}$$

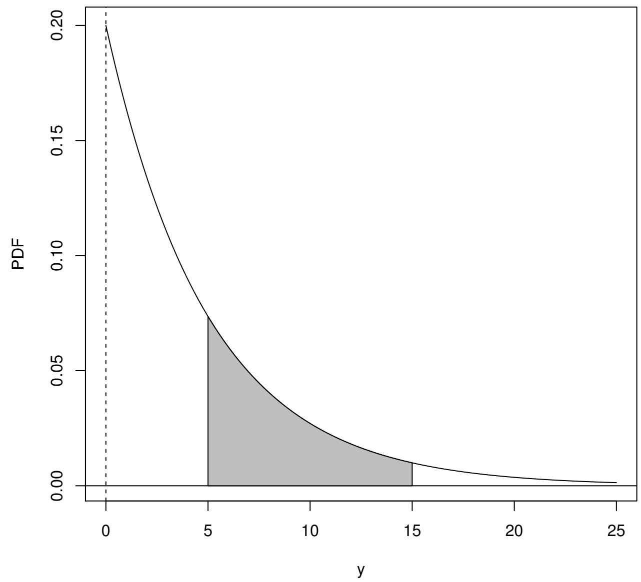
Question 2.
The crater radius $Y \sim \text{exponential}(\lambda = 0.2)$ meters.

(a) We have

$$\begin{aligned} P(5 < Y < 15) &= F_Y(15) - F_Y(5) \\ &= [1 - e^{-0.2(15)}] - [1 - e^{-0.2(5)}] \\ &= e^{-0.2(5)} - e^{-0.2(15)} \approx 0.3181. \end{aligned}$$

(b) See graph on the next page. The shaded area is $P(5 < Y < 15)$.
(c) The expected area is

$$E(W) = E(\pi Y^2) = \pi E(Y^2).$$



The easy way to do this problem is to remember that for an exponential distribution, $E(Y) = 1/\lambda = 5$ and $\text{var}(Y) = 1/\lambda^2 = 25$. Therefore,

$$E(Y^2) = \text{var}(Y) + [E(Y)]^2 = 25 + (5)^2 = 50$$

and $E(W) = 50\pi \approx 157.08$ (meters)^2.

If you did not remember $E(Y)$ and $\text{var}(Y)$ for an exponential distribution, just calculate $E(Y^2)$ directly; i.e.,

$$E(Y^2) = \int_0^\infty y^2 \times 0.2e^{-0.2y} dy = 50.$$

This integral can be done using integration by parts (twice).

Question 3.

(a) The median $\phi_{0.5}$ solves

$$P(T \leq \phi_{0.5}) = F_T(\phi_{0.5}) = 1 - e^{-\left(\frac{\phi_{0.5}}{3.216}\right)^{2.917}} \stackrel{\text{set}}{=} 0.5.$$

From this equation, we have

$$\begin{aligned} e^{-\left(\frac{\phi_{0.5}}{3.216}\right)^{2.917}} = 0.5 \quad &\Longrightarrow \quad \left(\frac{\phi_{0.5}}{3.216}\right)^{2.917} = -\ln(0.5) \\ &\Longrightarrow \quad \frac{\phi_{0.5}}{3.216} = [-\ln(0.5)]^{1/2.917} \\ &\Longrightarrow \quad \phi_{0.5} = 3.216[-\ln(0.5)]^{1/2.917} \approx 2.84 \text{ years.} \end{aligned}$$

(b) The values $\widehat{\beta} \approx 2.917$ and $\widehat{\eta} \approx 3.216$ are estimates of the population parameters β and η based on a single random sample (i.e., the 25 cooling units observed). There is variation associated with these estimates because we do not observe the entire population of cooling units. Therefore, each estimate's standard error measures how variable it is.
(c) The hazard function is increasing which means the population of cooling units is getting weaker over time. In other words, the rate of cooling unit failure increases over time (and it does so at an increasing rate because the function is concave up).

Question 4.
(a) We are 90 percent confident that the population mean beam breaking strength μ is between 7035.15 and 7372.56 lb/in^2.
(b) It would be longer than the corresponding 90 percent confidence interval. A prediction interval is designed to capture the value of a single individual from the population (here, a beam). Doing so is much more variable than trying to estimate the mean of a population distribution. Therefore, the margin of error associated with the interval will be larger; hence, making the interval longer.
(c) A qq plot is formed by plotting the observed data (in order) versus the ordered quantiles from an assumed probability distribution. If these quantities "agree" with each other to a large degree, this will result in the qq plot displaying a linear trend. This is why linearity in the qq plot supports the distribution in question. On the other hand, if the observed data and quantiles "disagree" with each other, this will result in the qq plot showing nonlinear patterns.

With only 30 observations, it is a tall order to make definite prognostications regarding the population distribution of the beam strengths. Only the Oracle knows the true population distribution. However, based on the sample we have, the normal distribution would at least not be refuted.

Question 5.
(a) We are 95 percent confident that the difference in the population proportions $p_1 - p_2$ is between -0.175 and 0.003. This interval does include the value "0," albeit barely, so we would conclude (at the 95 percent confidence level) that there is no difference between the population proportion of individuals who experience sleep problems/insomia for the two groups of smartphone users.
(b) It should not bother you. A 90 percent confidence interval has less confidence, so it will necessarily be a shorter interval. It is interesting that our conclusion now (i.e., at the 90 percent confidence level) would be that there *is* a population-level difference between the heavy and non-heavy groups. However, there is a trade-off; we have declared there is a difference, but we have less confidence in this assertion.
(c) We could write a confidence interval for the difference of the population means $\mu_1 - \mu_2$, where

$$\begin{aligned} \mu_1 &= \text{population mean number of steps taken per day for "non-heavy" group} \\ \mu_2 &= \text{population mean number of steps taken per day for "heavy" group.} \end{aligned}$$

I would probably choose the confidence interval for $\mu_1 - \mu_2$ that uses an unequal variance assumption (based on the boxplots, which display a fairly noticeable difference in variability between the two groups). One could formally check this assumption by writing a confidence interval for the ratio of the population variances.

If the confidence interval for $\mu_1 - \mu_2$ included "0," this would suggest that there is no difference between the population means between the two groups. We should check the normality assumption with both samples, but remember that population level inferences for means are robust to normality departures, and with large sample sizes (> 100), normality might matter even less (because of the Central Limit Theorem).

Question 6.
(a) This is a matched pairs study because each patient is observed under both experimental conditions (without herbal medicine and with herbal medicine).
(b) We are 95 percent confident that the population mean difference $\mu_1 - \mu_2$ is between -1.81 and -1.24 hours. Because this interval excludes zero and includes only negative values, we can say (with 95 percent confidence) that the population mean number of hours of sleep per night for individuals not taking the herbal medicine is less that the population mean number of hours of sleep per night for individuals taking the herbal medicine.
(c) If pairing through the matched pairs strategy was effective at reducing the variation among the individuals observed under different experimental conditions (with medicine/without), then the two-independent-sample confidence interval for $\mu_1 - \mu_2$ would be wider, and potentially much wider. The advantage of pairing is that it removes the variation arising from observing different individuals under different experimental conditions. This results in more precise inference.

On the other hand, if pairing the different treatments on the same individual did not remove this source of variability (or reduced it only very mildly), then the matched pairs and two-independent-sample confidence intervals might be very similar.

Question 7.
(a) There is strong evidence that temperature and the level of dissolved oxygen are linearly related in the population. A 95 percent confidence interval for β_1 is $(-0.3963, -0.2265)$, which excludes zero. Also, the probability value (p-value) for the test of

$$\begin{gathered} H_0 : \beta_1 = 0 \\ \text{versus} \\ H_1 : \beta_1 \neq 0 \end{gathered}$$

is 0.0000000003, which is incredibly small. This is overwhelming evidence against H_0, and it gives us an indication just how far away from zero the confidence interval actually is.
(b) About 42.2 percent of the variability of the dissolved oxygen level data is explained by the simple linear regression model that includes temperature. The remaining 57.8 percent is explained by other sources.
(c) For the population of water specimens with temperature of 15 deg C, we are 95 percent confident that the population mean level of dissolved oxygen is between 7.0431 and 7.4245 mg/L.

Question 8.

(a) The design matrix $\mathbf{X}$ has 17 rows and 4 columns. Here are the first two rows:

$$\mathbf{X} = \begin{pmatrix} 1 & 35 & 110 & 160 \\ 1 & 25 & 130 & 180 \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix}_{17\times 4}$$

(b) The overall F statistic tests

$$H_0: \beta_1 = \beta_2 = \beta_3 = 0$$
$$\text{versus}$$
$$H_1: \text{at least one } \beta_j \text{ is not equal to } 0.$$

With a probability value of 0.1585, we do not have strong evidence against H_0. Therefore, our conclusion is that none of the three variables (temperature, moisture, and speed) are linearly related to SDFP in the population.

(c) The model with the quadratic terms fits the data much better. Note that the overall F test in the quadratic model is strongly significant (p-value = 0.0000005) and each variable is now strongly related to SDFP (seen through the individual p-values). Note that R^2 is much higher in the quadratic model (0.9699 versus 0.3197). Also, the residual standard error is much smaller in the quadratic model (0.2248 versus 0.9376). All of this is evidence in favor of the quadratic model.

Question 9.

(a) The `geometry` and `angle` main effects are strongly significant (seen through the very small p-values). On the other hand, the main effect of `speed` is not significant (p-value = 0.88).

(b) The interaction effect of `speed:angle` is significant. The p-value for the test of this effect is 0.0011 and the interaction plot produces lines that actually cross. The other two two-way interaction effects (`speed:geometry` and `geometry:angle`) are not significant in the population.

(c) The residual SS increases by this amount:

$$16.67 + 48.17 + 28.17 = 93.00 \quad \text{(up to rounding error)}$$

These are the 3 sums of squares associated with `speed:geometry`, `geometry:angle`, and the three-way interaction `speed:geometry:angle`. When you fit the smaller model excluding these terms, their SS from the full model get absorbed into the residual SS for the smaller model.

(d) The residual plot shows no structural patterns; i.e., it looks totally random. This is supportive of the model because it does not reveal violations of the underlying statistical assumptions.