1. A manufacturer buys 60 percent of a raw material (e.g., nails) from Supplier 1 and 40 percent from Supplier 2. There are no other suppliers used by the manufacturer. Two percent of the raw materials from Supplier 1 are defective. For Supplier 2, four percent of the raw materials are defective. A piece of raw material is selected from the production line at random.

(a) Define two relevant events A and B using the information above. Interpret each percent above as a probability (two conditional; two not conditional).
(b) What is the probability the raw material selected is defective? What "law" are you using here?
(c) If the raw material selected was not defective, what is the probability it came from Supplier 2?

2. The South Carolina Department of Revenue estimates that 10 percent of all individual state income tax forms filed during 2016 will contain "serious errors."

(a) Conceptualizing each tax form filed as a "trial," state the 3 Bernoulli trial assumptions in the context of this problem. Assume these assumptions are true for the parts below.
(b) If an auditor processes 20 tax forms, calculate the probability that at least 2 will contain serious errors.
(c) An auditor processes tax forms until he finds the first one with serious errors. What is the probability he will process fewer than 3 forms?

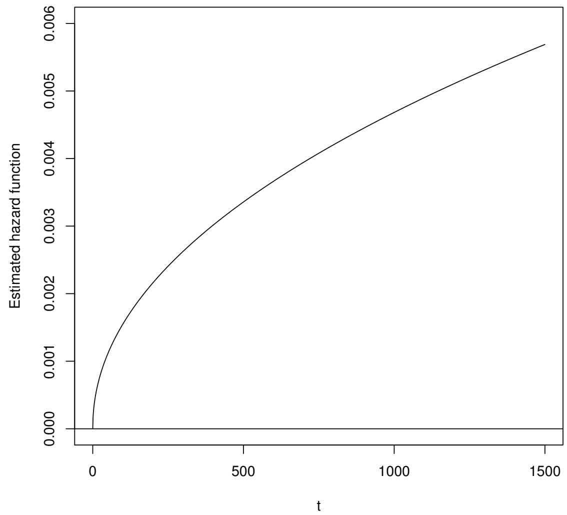
3. Let Y represent the number of requests for assistance received by a towing service per hour. Suppose Y follows a Poisson distribution with mean $\lambda = 5$.

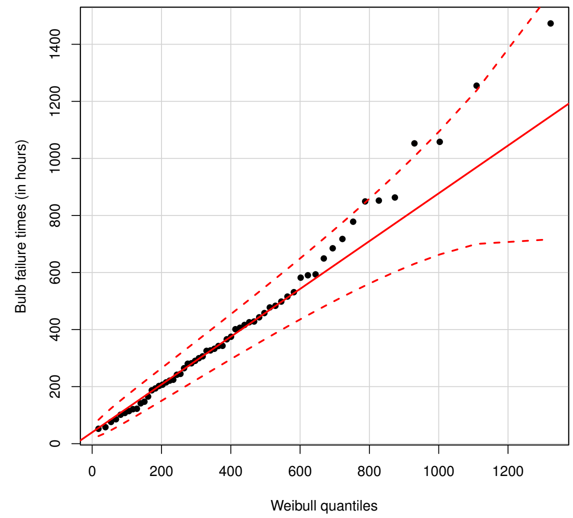
(a) Hourly company revenue R (in dollars) has been modeled as a quadratic function of Y; specifically, $R = -120 + 40Y + 10Y^2$. Calculate the expected hourly revenue $E(R)$.
(b) Let W denote the time it takes for the service to receive its first call. Name the distribution of W and find the probability the service will have to wait longer than 10 minutes to receive the first call. **Note:** 10 minutes is 1/6th of an hour.

4. A mechanic records Y, the amount of time during a one-hour period that a machine operates at its maximum capacity. Values of Y are between 0 and 1; that is, "0" means the machine never operates at maximum capacity during the hour and "1" means the machine operates at full capacity during the entire hour. The random variable Y is continuous and has the following probability density function (pdf)

$$f_Y(y) = \begin{cases} 4y(1-y^2), & 0 \leq y \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Calculate $E(Y)$ and interpret what this means in words.
(b) Let $F_Y(y)$ denote the cumulative distribution function (cdf) of Y. Calculate $F_Y(0.5)$.





5. A light bulb company manufactures filaments that are not expected to wear out during an extended period of "intense use." With the goal of guaranteeing bulb reliability in these conditions, engineers at the company sample $n = 60$ bulbs, simulate their long-term intense use, and record T, the hours until failure for each bulb. Here are the lifetimes they observed (i.e., hours until failure):

443.0	593.5	374.9	582.0	590.4	290.4	264.6	649.3	531.1	849.2
101.5	107.7	141.5	342.4	122.5	401.3	57.9	147.2	281.9	852.2
52.3	477.6	85.7	221.1	685.0	343.1	187.1	515.7	202.3	1058.0
498.2	241.6	244.7	1052.7	406.4	165.2	193.7	425.7	76.2	416.2
457.9	778.1	483.4	224.2	325.4	1254.9	280.3	206.8	717.6	863.0
327.0	332.7	214.9	121.0	428.3	306.2	1473.1	365.9	114.3	299.7

The engineers assume a Weibull(β, η) model for T. Here are the maximum likelihood estimates from fitting the model to the data:

$$\widehat{\beta} = 1.48 \qquad \widehat{\eta} = 459.2.$$

(a) Based on the Weibull model fit, ten percent of the bulbs will last longer than what value?
(b) The estimate of β is larger than 1, which means the estimated hazard function $h_T(t)$ is an increasing function of t. I have plotted this function on this page (top, left). Interpret what it means.
(c) The quantile-quantile (qq) plot from the Weibull model fit is also shown above (right). Interpret what information is revealed by this plot. Should the engineers be concerned about using the Weibull model to quantify the reliability of these light bulbs?

Binomial:

$$p_Y(y) = \begin{cases} \binom{n}{y} p^y (1-p)^{n-y}, & y = 0, 1, 2, ..., n \\ 0, & \text{otherwise.} \end{cases}$$

Geometric:

$$p_Y(y) = \begin{cases} (1-p)^{y-1} p, & y = 1, 2, 3, ... \\ 0, & \text{otherwise.} \end{cases}$$

Negative binomial:

$$p_Y(y) = \begin{cases} \binom{y-1}{r-1} p^r (1-p)^{y-r}, & y = r, r+1, r+2, ... \\ 0, & \text{otherwise.} \end{cases}$$

Hypergeometric:

$$p_Y(y) = \begin{cases} \dfrac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}, & y \leq r \text{ and } n - y \leq N - r \\ 0, & \text{otherwise.} \end{cases}$$

Poisson:

$$p_Y(y) = \begin{cases} \dfrac{\lambda^y e^{-\lambda}}{y!}, & y = 0, 1, 2, ... \\ 0, & \text{otherwise.} \end{cases}$$

Exponential:

$$f_Y(y) = \begin{cases} \lambda e^{-\lambda y}, & y > 0 \\ 0, & \text{otherwise.} \end{cases} \qquad F_Y(y) = \begin{cases} 1 - e^{-\lambda y}, & y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Gamma:

$$f_Y(y) = \begin{cases} \dfrac{\lambda^\alpha}{\Gamma(\alpha)} y^{\alpha-1} e^{-\lambda y}, & y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Normal (Gaussian):

$$f_Y(y) = \begin{cases} \dfrac{1}{\sqrt{2\pi}\sigma} e^{-(y-\mu)^2/2\sigma^2}, & -\infty < y < \infty \\ 0, & \text{otherwise.} \end{cases}$$

Weibull:

$$f_T(t) = \begin{cases} \dfrac{\beta}{\eta}\left(\dfrac{t}{\eta}\right)^{\beta-1} e^{-(t/\eta)^\beta}, & t > 0 \\ 0, & \text{otherwise.} \end{cases} \qquad F_T(t) = \begin{cases} 1 - e^{-(t/\eta)^\beta}, & t > 0 \\ 0, & \text{otherwise.} \end{cases}$$