Question 1.
(a) Define the events

$$\begin{aligned} A &= \{\text{manufacturer buys material from Supplier 1}\} \\ B &= \{\text{material is defective}\}. \end{aligned}$$

We are given that $P(A) = 0.6$, $P(\overline{A}) = 0.4$, $P(B|A) = 0.02$, and $P(B|\overline{A}) = 0.04$.
(b) With the definitions above, we want to compute $P(B)$. Use the Law of Total Probability:

$$\begin{aligned} P(B) &= P(B|A)P(A) + P(B|\overline{A})P(\overline{A}) \\ &= 0.02(0.6) + 0.04(0.4) = 0.028. \end{aligned}$$

(c) With the definitions above, we want to compute $P(\overline{A}|\overline{B})$. Using the definition of conditional probability:

$$\begin{aligned} P(\overline{A}|\overline{B}) &= \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})} \\ &= \frac{P(\overline{B}|\overline{A})P(\overline{A})}{1 - P(B)} = \frac{[1 - P(B|\overline{A})]P(\overline{A})}{1 - P(B)} = \frac{(1 - 0.04)(0.4)}{1 - 0.028} \approx 0.395. \end{aligned}$$

The second equality above arises from using the multiplication rule in the numerator and using the complement rule in the denominator.

Question 2.
(a) Here are the Bernoulli trial assumptions for this problem:

1. Each form filed either contains serious errors or it does not.
2. Each form filed is independent.
3. The probability that a form filed has serious errors is $p = 0.1$, and this probability is the same for each form filed.

(b) Let Y denote the number of forms that contain serious errors (out of $n = 20$). Then $Y \sim b(n = 20, p = 0.1)$. We want

$$\begin{aligned} P(Y \geq 2) &= 1 - P(Y = 0) - P(Y = 1) \\ &= 1 - \binom{20}{0}(0.1)^0(1 - 0.1)^{20} - \binom{20}{1}(0.1)^1(1 - 0.1)^{19} \\ &= 1 - (0.9)^{20} - 20(0.1)(0.9)^{19} \approx 0.608. \end{aligned}$$

b) Let W denote the number of forms needed to find the first one with serious errors. Then $W \sim \text{geom}(p = 0.1)$. We want

$$\begin{aligned} P(W < 3) &= P(W = 1) + P(W = 2) \\ &= (1 - 0.1)^0(0.1) + (1 - 0.1)^1(0.1) \\ &= 0.1 + 0.9(0.1) = 0.19. \end{aligned}$$

Question 3.
Recall that for $Y \sim \text{Poisson}(\lambda = 5)$, we have $E(Y) = \text{var}(Y) = 5$. Therefore,

$$E(Y^2) = \text{var}(Y) + [E(Y)]^2 = 5 + 5^2 = 30.$$

This calculation is needed below. We have

$$\begin{aligned} E(R) &= E(-120 + 40Y + 10Y^2) \\ &= -120 + 40E(Y) + 10E(Y^2) \\ &= -120 + 40(5) + 10(30) = 380. \end{aligned}$$

The expected hourly revenue is 380 dollars.
(b) The time to the first occurrence in a Poisson process follows an exponential distribution. Therefore, $W \sim \text{exponential}(\lambda = 5)$. We want to calculate $P(W > 1/6)$. We have

$$\begin{aligned} P(W > 1/6) = 1 - P(W \leq 1/6) &= 1 - \int_0^{1/6} 5e^{-5w} dw \\ &= 1 - 5\left[\left(-\frac{1}{5}\right) e^{-5w}\Big|_0^{1/6}\right] \\ &= 1 + (e^{-5/6} - 1) = e^{-5/6} \approx 0.435. \end{aligned}$$

Question 4.
(a) We have

$$\begin{aligned} E(Y) &= \int_0^1 y \times 4y(1-y^2)dy \\ &= 4\int_0^1 (y^2 - y^4)dy \\ &= 4\left(\frac{y^3}{3} - \frac{y^5}{5}\right)\Big|_0^1 = 4\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{8}{15} \approx 0.533. \end{aligned}$$

This is the value of Y we would expect to observe "on average." Equivalently, it is the center of gravity of the pdf $f_Y(y)$; i.e., where $f_Y(y)$ would balance.
(b) Recall that $F_Y(y) = P(Y \leq y)$. Therefore,

$$\begin{aligned} F_Y(0.5) = P(Y \leq 0.5) &= \int_0^{0.5} 4y(1-y^2)dy \\ &= 4\int_0^{0.5} (y - y^3)dy \\ &= 4\left(\frac{y^2}{2} - \frac{y^4}{4}\right)\Big|_0^{0.5} = 4\left[\frac{(0.5)^2}{2} - \frac{(0.5)^4}{4}\right] = 0.4375. \end{aligned}$$

Question 5.
(a) We want to find the 90th percentile; i.e., the 0.9 quantile $\phi_{0.9}$. We are left to solve

$$\begin{aligned} F_T(\phi_{0.9}) &= 1 - e^{-(\phi_{0.9}/459.2)^{1.48}} \stackrel{\text{set}}{=} 0.9 \\ &\Longrightarrow e^{-(\phi_{0.9}/459.2)^{1.48}} = 0.1 \\ &\Longrightarrow -(\phi_{0.9}/459.2)^{1.48} = \ln(0.1) \\ &\Longrightarrow \frac{\phi_{0.9}}{459.2} = [-\ln(0.1)]^{1/1.48} \implies \phi_{0.9} = 459.2[-\ln(0.1)]^{1/1.48} \approx 806.76. \end{aligned}$$

Therefore, about 10 percent of the bulbs will last longer than 806.76 hours.
(b) An increasing hazard function means that the rate of bulb failure increases with time; i.e., the population of bulbs gets weaker over time.
(c) The qq plot shows pretty good agreement between the observed data and the corresponding Weibull quantiles overall. This agreement is very strong early on (i.e., for smaller times). There is a little departure in the upper tail (i.e., the observed times are a little larger than would be expected under the Weibull model), but nearly all of these observations do fall within the bands of uncertainty. I think a Weibull model is reasonable here−it is certainly not grossly unreasonable.

Given that the Weibull assumption is reasonable, the engineers could feel confident in using this model to quantify the reliability of these bulbs.