Question 1.
(a) Let Y denote the number of rental properties (out of $n = 6$) that have photoelectric smoke detectors. Under appropriate Bernoulli trial assumptions; see part (b), $Y \sim b(n = 6, p = 0.8)$, that is, a binomial distribution. Therefore,

$$\begin{aligned} P(Y \geq 5) &= P(Y = 5) + P(Y = 6) \\ &= \binom{6}{5}(0.8)^5(0.2)^1 + \binom{6}{6}(0.8)^6(0.2)^0 \approx 0.393 + 0.262 = 0.655. \end{aligned}$$

(b) The Bernoulli trial assumptions in this problem are

1. Each rental property either has photoelectric smoke detectors or not.
2. The probability of having photoelectric smoke detectors (0.8) is the same for every rental property.
3. The rental properties are independent.

(c) The probability a rental property does not have photoelectric smoke detectors is $1-0.8 = 0.2$. The number of rental properties inspected to find the first one without photoelectric smoke detectors follows a geometric distribution with probability 0.2.

Question 2.
(a) The desired probability is

$$P(Y > 3) = \int_3^\infty \frac{2}{y^3} dy = 2\left(-\frac{1}{2}y^{-2}\right)\Big|_3^\infty = \frac{1}{9} - \lim_{y\to\infty}\frac{1}{y^2} = \frac{1}{9} - 0 \approx 0.111.$$

This is the same answer you would get if you calculated

$$1 - P(Y \leq 3) = 1 - \int_1^3 \frac{2}{y^3} dy.$$

(b) The expected value of Y is

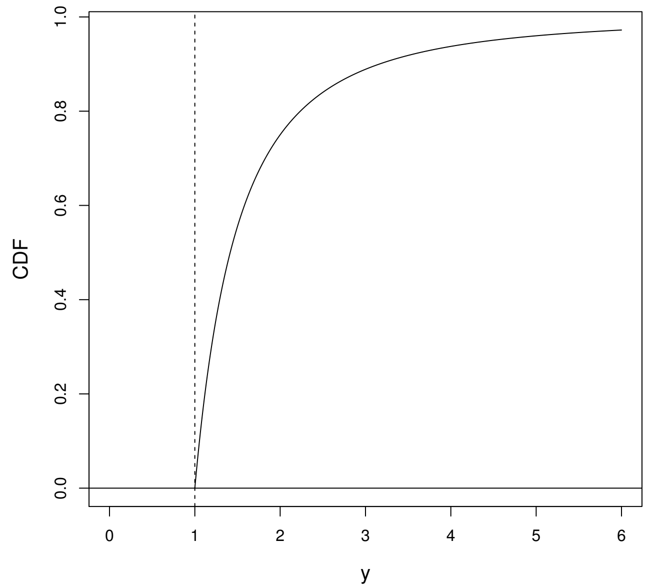
$$E(Y) = \int_1^\infty y \times \frac{2}{y^3} dy = \int_1^\infty \frac{2}{y^2} dy = 2\left(-y^{-1}\right)\Big|_1^\infty = 2\left(1 - \lim_{y\to\infty}\frac{1}{y}\right) = 2 \text{ seconds}.$$

Interpretation? Any of these are fine:

- $E(Y) = 2$ is the balance point of the pdf $f_Y(y)$; i.e., where the pdf would balance.
- For two cars randomly selected, the expected time headway is 2 seconds.
- Over many pairs of cars observed, the average time headway would be close to 2 seconds.

(c) I will derive the cumulative distribution function (cdf) formula. For $y \leq 1$, the cdf is

$$F_Y(y) = P(Y \leq y) = \int_{-\infty}^{y} 0 dt = 0.$$



For $y > 1$, the cdf is

$$F_Y(y) = P(Y \leq y) = \int_1^y \frac{2}{t^3} dt = 2\left(-\frac{1}{2}t^{-2}\right)\Bigg|_1^y = 1 - \frac{1}{y^2}.$$

This function is graphed above. You wouldn't have to derive the cdf formula to sketch a good graph. Examine the graph of the pdf and then sketch a function that describes $P(Y \leq y)$; i.e., a function describing how probability (area) cumulates as you traverse along the horizontal axis.

Question 3.
(a) Exponential, with parameter $\lambda = 1/\eta$. This can be seen by examining the Weibull cdf (or pdf) and substituting in $\beta = 1$. The hazard function under the exponential model is a constant function of time t. Therefore, the rate of healing would be constant over time.
(b) The median $\phi_{0.5}$ is the 0.5 quantile. We are left to solve

$$\begin{aligned} F_T(\phi_{0.5}) = P(T \leq \phi_{0.5}) &= 1 - e^{-(\phi_{0.5}/191)^1} \stackrel{\text{set}}{=} 0.5 \\ &\Longrightarrow e^{-\phi_{0.5}/191} = 0.5 \\ &\Longrightarrow -\frac{\phi_{0.5}}{191} = \ln(0.5) \Longrightarrow \phi_{0.5} = -191\ln(0.5) \approx 132.4 \text{ days.} \end{aligned}$$

(c) The Weibull model does not fit the healing times data very well. There is substantial deviation from linearity in the plot. Observed times between roughly 300-600 days are consistently larger than they should be under the Weibull model (even outside the bands of uncertainty).
(d) Lognormal, gamma, etc.

Question 4.
(a) There is no "right" answer here, but any answer that refers to a larger collection of concrete

beams is fine. This might be all concrete beams that are produced by a certain manufacturer, all beams that could be selected for a construction job, etc.
(b) The standard error of the sample mean is estimated by $s/\sqrt{27}$. The margin of error is estimated by $t_{26,0.025} \times s/\sqrt{27}$.
(c) The sampling distribution is t with degrees of freedom $n-1 = 26$. This requires the following assumptions:

1. The sample of beams is a random sample.
2. The flexural strength of the beams Y follows a normal distribution.

(d) It is certainly *not* consistent with the population mean μ being equal to 7 MPa. If $\mu = 7$, then the t statistic should be around zero because the sample mean $\overline{y}$ estimates the population mean μ correctly on average (i.e., it is unbiased).

If $t \approx 3.57$, then the numerator $\overline{y} - 7$ must be positive (because the denominator is positive); i.e., $\overline{y} > 7$. Because $\overline{y}$ estimates the population mean μ correctly on average, the observation $\overline{y} > 7$ is more consistent with $\mu > 7$. It is certainly *not* consistent with $\mu < 7$.

Question 5.
(a) This is a matched pairs design; i.e., a measurement is made on each punter under two different experimental conditions (air and helium). Therefore, the samples are dependent; not independent.
(b) We are 95 percent confident that the difference of the population mean punting distances $\mu_1 - \mu_2$ is between -2.72 and -0.63 yards. Because this interval does not contain zero and includes only negative values, this is consistent with the population mean punting distance for air-filled footballs (μ_1) being smaller than the population mean punting distance for helium-filled footballs (μ_2).
(c) The confidence interval in part (b) arises from acknowledging the matched-pairs aspect of the experiment. In a matched-pairs design, variation is reduced because we are observing punting distances for the two types of footballs on the same punter. In the independent two-sample design, we must contend with the extra variation that arises from having one punter punt an air-filled ball and a different punter punt a helium-filled ball. There is more variation with this design because of the inherent differences between the two-punters. The interval in part (b) is shorter because this source of variation is not present in the matched-pairs design.

Question 6.
(a) If H_0 was true, then we would expect to see $F \approx 1$. However, this is not the case here. In fact, the tail probability to the right of F is approximately 0.004, which is very small. This means F is way out in the right tail of its probability distribution, more consistent with H_1.
(b) We are (at least) 95 percent confident that the difference of the population mean ALP levels $\mu_2 - \mu_1$ is between -5.15 and 62.33 IU/L. Because the interval contains zero, we cannot say that the population mean ALP levels are different between children taking phenobarbital and the control group.
(c) The Tukey pairwise interval in part (b) is one of 6 intervals that have been constructed to have a "familywise" confidence level of 95%. This means the confidence level of each of the 6 individual intervals is higher than 95%. Therefore, the Tukey confidence interval will be wider.

(d) The ANOVA procedure compares means by formulating two unbiased estimators of the common population variance σ^2 when H_0 is true. If the difference between these estimates (i.e., MS_{trt} and MS_{res}) is large, then H_0 will be rejected. Otherwise F will be around 1 and H_0 will not be rejected.

Question 7.

(a) There is no "right" answer here, but any answer that refers to a larger collection of scamp grouper fish is fine. This might be all scamp grouper fish in the Gulf of Mexico or all scamp grouper fish off the Florida coast.

(b) Yes, it does. The 95% confidence interval for β_1 excludes zero and includes only positive values. This is consistent with mercury concentration Y and length x being (positively) linearly related in the population.

(c) This means 68.3% of the variation in the mercury concentration data is explained by the linear relationship with length. The other 31.7% of the variation is explained by other factors. Remember that R^2's interpretation is only valid under the assumption that the simple linear regression model is correct. If Y and x are related in some other way (e.g., quadratic, etc.), then R^2's interpretation is compromised.

(d) The least squares regression equation evaluated at $x = 100$ is

$$\widehat{Y} = -0.30733 + 0.00104(100) = -0.20333 \text{ mg/kg}.$$

This estimate makes no sense as a mercury concentration cannot be negative. Examining the data, we see the range of the fish lengths observed in the study was about 300-700mm. Therefore, using the least-squares regression equation to estimate the mercury concentration when $x = 100$ is an extrapolation.

Question 8.

(a) The least squares regression equation evaluated at these x values is

$$\widehat{Y} = 2244.923 + 28.925(18.69) + 7.644(15.65) + 4.297(45.01) - 37.354(58.21) \approx 924.19 \text{ kcal/kg}.$$

This is the predicted value. The residual is

$$e = Y - \widehat{Y} = 947 - 924.19 \approx 22.81.$$

(b) They are not. The probability value 0.1002 is used to assess the linear relationship between energy Y and the `garbage` percentage x_3 after the first two variables (`plastic` percentage and `paper` percentage) have been added to the model. On the other hand, the confidence interval for β_3 (the linear effect of `garbage`) is interpreted conditional on all 3 other independent variables being in the model (i.e., `plastic` percentage, `paper` percentage, and `moisture` percentage). In other words, these two inference techniques provide insight on different questions.

(c) The residual plot looks fairly random in appearance. This suggests no obvious model deficiencies. If this plot had contained systematic patterns (e.g., increasing variation, quadratic patterns, etc.), then that would suggest model flaws. This is not the case here.

Question 9.

(a) The null hypothesis H_0 says the population mean fracture toughness is the same for each of the 4 treatment combinations. The alternative hypothesis H_1 says that the population means are different somehow (although it does not specify how the population means are different). In symbols,

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$$
$$\text{versus}$$
$$H_1 : \text{the population means } \mu_i \text{ are not all equal.}$$

Here μ_1, μ_2, μ_3, and μ_4 denote the population mean fracture toughness for the four treatment combinations, respectively.

(b) In the factorial analysis, the treatment sum of squares SS_{trt} from the one way ANOVA is split into the sums of squares for the main effects and the interaction effects. Therefore, $\text{SS}_A + \text{SS}_B + \text{SS}_{AB} = 16.2625$.

(c) An interaction plot that crosses is strong visual evidence of interaction between the two factors A and B (i.e., mixture type and temperature). Interaction means the way the response Y (fracture toughness) is related to one factor (e.g., mixture type) changes depending on what level of the other factor (e.g., temperature) you are at.

(d) With two additional factors C and D, this would be a 2^4 experiment. One replicate of a 2^4 experiment would require 16 asphalt specimens. Three replicates would require 48.