Note: This homework assignment covers Chapter 2.

1. A medical professional observes adult male patients entering an emergency room. She classifies each patient according to his blood type (AB^+, AB^-, A^+, A^-, B^+, B^-, O^+, and O^-) and whether his systolic blood pressure (SBP) is low, normal, or high.

(a) Find the number of ways a patient can be classified according to his blood type and SBP level, write out each way, and refer to this as the sample space for the next male patient to be observed.
(b) Assign probabilities to each of the outcomes in your sample space. There is no "right" way to do this; perhaps find reliable references online regarding how prevalent blood types are (for American males) and also for low/normal/high blood pressure. Cite your sources and be able to defend your answers. Of course, make sure your probabilities add to 1.
(c) Using your sample space and the probabilities you assigned, answer the following questions. Show explicitly how you obtain your answers.

- What is the probability the next male has an AB blood type with normal SBP?
- What is the probability the next male has high SBP and does not have type O blood?
- What is the probability the next male has a blood type with a $^+$ rhesus status?

2. (a) Suppose $P(A) = 0.5$ and $P(A \cap \overline{B}) = 0.2$. If A and B are independent, find $P(B)$. Can you calculate $P(B)$ otherwise; i.e., if A and B are not independent?
(b) Suppose $P(A) = 0.6$, $P(B) = 0.4$, and $P(A \cup B) = 0.8$. Find $P(A|B)$ and $P(B|A)$. Are A and B independent?
(c) Suppose $P(A \cup B) = 0.5$, $P(A) = 2P(B)$, and $P(A|B) = P(A)$. Find $P(A)$, $P(B)$, and $P(B|A)$.

3. An oil exploration company currently has two active projects, one in Asia and one in Europe. Let A be the event that the one in Asia is successful and let B be the event that the one in Europe is successful. Seventy percent of past projects in Asia and forty percent of past projects in Europe have been successful, so take $P(A) = 0.7$ and $P(B) = 0.4$.

(a) For the two active projects, it is probably reasonable to assume that A and B independent. Why? Can you think of a scenario where A and B would not be independent? Whatever your answer is here, assume A and B are independent in the parts below.
(b) If the Asian project is not successful, what is the probability that the European project is also not successful? Explain your reasoning.
(c) What is the probability that at least one of the two projects will be successful?
(d) What is the probability that exactly one of the two projects will be successful?
(e) What is the probability that neither project will be successful?

4. Togolese scientists have identified a new strain of the H1N1 virus. The genetic sequence of the new strain consists of alterations in the hemagglutinin protein, making it significantly different than the usual H1N1 strain. Public health officials wish to study the population of residents in Lomé. Suppose that in this population,

- the probability of catching the usual strain is 0.10
- the probability of catching the new strain is 0.05
- the probability of catching both strains is 0.01.

(a) Find the probability of catching the usual strain or the new strain.
(b) Find the probability of catching the usual strain, given that the new strain is caught.
(c) Find the probability of catching the new strain, given that at least one strain is caught.
(d) Find the probability of not catching the usual strain, given that the new strain is not caught.

5. A large company is accustomed to training operators who do certain tasks on a production line. Those operators who take the training course are able to meet their production quotas 90 percent of the time. Operators who do not take the training course meet their quotas 60 percent of the time. Seventy percent of the operators have taken the training course.

(a) Define relevant events (there are 2) using event notation and interpret the three percentages above in terms of probabilities (using appropriate probability notation). Two of these are conditional probabilities.
(b) Find the probability that an operator has taken the training course and meets his/her production quota.
(c) Find the probability that an operator meets his/her production quota.
(d) Find the conditional probability that an operator has taken the training course, given that s/he meets his/her production quota.
(e) Are the two events you defined in part (a) independent? Explain.

6. Reliability engineers often work with systems having components connected in parallel. In this problem, we will interpret the phrase "in parallel" as follows: The system is reliable (i.e., it is functioning) if **at least one** of the components is functioning. As a frame of reference, consider a two-engine aircraft.

- If both engines are functioning, the aircraft is functioning (at least in lieu of non-engine related problems).
- If only one engine is functioning, the aircraft still functions (although losing one engine may warrant an immediate landing).

- If both engines are not functioning, the aircraft is not functioning.

In this problem, we will denote by n the number of components in a parallel system.

(a) Suppose $n = 2$. If the two components are functioning **independently**, each with probability p, show that the system reliability r_2 is given by

$$r_2 = 1 - (1 - p)^2.$$

Hint: Let $A_1 = \{\text{component 1 is functioning}\}$ and $A_2 = \{\text{component 2 is functioning}\}$. Then $r_2 = P(A_1 \cup A_2)$.

(b) Generalize the result in part (a) to consider a parallel system with n components (each functioning independently with probability p). That is, show that the system reliability is

$$r_n = 1 - (1 - p)^n.$$

Hint: $r_n = P(A_1 \cup A_2 \cup \cdots \cup A_n) = 1 - P(\overline{A}_1 \cap \overline{A}_2 \cap \cdots \cap \overline{A}_n)$. Now use the fact that the components are independent.

(c) I have a parallel system with $n = 5$ components (each functioning independently with probability p). How unreliable can the individual components be and still have a system with reliability $r_5 = 0.9999$?

(d) So far in this problem, we have made two critical assumptions:

(A1) the components function independently

(A2) the components each function with the same probability p.

Give a real-life example where Assumption (A1) is likely violated. Give a real-life example where Assumption (A2) is likely violated. Do not use the plane example I used at the outset. Explain your examples sufficiently so I can understand.