Stat 509 Homework 1 Solution

1. (a)

S={LowSBP AB+, NormalSBP AB+, HighSBP AB+,
LowSBP AB-, NormalSBP AB-, HighSBP AB-,
LowSBP A+, NormalSBP A+, HighSBP A+,
LowSBP A-, NormalSBP A-, HighSBP A-,
LowSBP B+, NormalSBP B+, HighSBP B+,
LowSBP B-, NormalSBP B-, HighSBP B-,
LowSBP O+, NormalSBP O+, HighSBP O+,
LowSBP O-, NormalSBP O-, HighSBP O-}

n_s =8*3=24

(b)

We assign probabilities to Blood type and Blood pressure level for American male.

Blood type	AB+	AB-	A+	A-	B+	B-	O+	O-
Probability	0.08	0.005	0.275	0.01	0.31	0.01	0.29	0.02

SBP	Low (<90)	Normal (90~140)	High (>140)
Probability	0.001	0.715	0.284

Assume Blood type and SBP level are independent, then

Probability	AB+	AB-	A+	A-	B+	B-	O+	O-
Low	0.00008	0.000005	0.000275	0.00001	0.00031	0.00001	0.00029	0.00002
Normal	0.05720	0.003575	0.196625	0.00715	0.22165	0.00715	0.20735	0.01430
High	0.02272	0.00142	0.07810	0.00284	0.08804	0.00284	0.08236	0.00568

Sources links:

http://www.bloodpressureuk.org/BloodPressureandyou/Thebasics/Bloodpressurechart

https://obs.health.nokia.com/us/bp

https://pdfs.semanticscholar.org/a929/2b5e6f297fbc591fcf221ab02c29cd05db0b.pdf

(c) P(Normal SBP and AB)

= P(Normal SBP and AB+) + P(Normal SBP and AB−)

= 0.0572 + 0.003575

= 0.060775

P(High SBP and $\overline{O}$)

= P(High SBP and AB+) + P(High SBP and AB−)

+ P(High SBP and A+) + P(High SBP and A−)

+ P(High SBP and B+) + P(High SBP and B−)

= 0.02272 + 0.00142 + 0.07810 + 0.00284 + 0.08804 + 0.00284

= 0.19596

P(+ rhesus status)

= P(AB+) + P(A+) + P(B+) + P(O+)

= 0.08 + 0.275 + 0.31 + 0.29

= 0.955

2. (a) $P(A \cap \bar{B}) = P(A) \cdot P(\bar{B}) = 0.2$

$$\Rightarrow \quad P(\bar{B}) = 0.2 / P(A) = 0.2/0.5 = 0.4$$

$$P(B) = 1 - P(\bar{B}) = 1 - 0.4 = 0.6$$

No, we can't calculate $P(B)$ if A and B are not independent.

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.8$

$$0.6 + 0.4 - P(A \cap B) = 0.8$$

$$P(A \cap B) = 0.2$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.4} = 0.5$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.6} = \frac{1}{3}$$

No. A and B are not independent, since $P(A|B) \neq P(A)$.
$0.5 \neq 0.6$

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5$

plug in $P(A) = 2P(B)$, $\Rightarrow 2P(B) + P(B) - P(A \cap B) = 0.5$

$$\Rightarrow \quad 3P(B) - P(A \cap B) = 0.5$$

Since $P(A|B) = P(A)$, A and B are independent, we have $P(A \cap B) = P(A) \cdot P(B)$

$$\Rightarrow \quad 3P(B) - P(A) \cdot P(B) = 0.5$$

$$3P(B) - 2P(B) \cdot P(B) = 0.5$$

$$P(B) = \frac{3 \pm \sqrt{5}}{2}$$

$$\approx 0.382 \text{ or } 2.618$$

since $0 \leq P(B) \leq 1$, we get $P(B) = 0.382$.

3. (a) We could assume A and B are independent, since one is in Asia and one in Europe.

However, if those two projects are done using the same technic or managed by the same group of people, A and B would not be independent.

(b) Since we assumed that A and B are independent,

$P(\bar{B} \mid \bar{A}) = P(\bar{B}) = 1 - P(B) = 1 - 0.4 = 0.6$.

(c) P(at least one will be successful)

$= P(A \cap B) + P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$= P(A) \cdot P(B) + P(A) \cdot P(\bar{B}) + P(\bar{A}) \cdot P(B)$

$= 0.7 \times 0.4 + 0.7 \times 0.6 + 0.3 \times 0.4$

$= 0.28 + 0.42 + 0.12$

$= 0.82$

(d) P(exactly one project will be successful)

$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$= 0.7 \times 0.6 + 0.3 \times 0.4$

$= 0.54$

(e) P (neither project will be successful)

$= P(\bar{A} \cap \bar{B})$

$= 0.3 \times 0.6$

$= 0.18$

4. (a) A = catching the usual strain

B = catching the new strain

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.1 + 0.05 - 0.01$$
$$= 0.14$$

(b)
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{0.01}{0.05}$$
$$= 0.2$$

(c) $P(B \mid A \cup B)$
$$= \frac{P(B \cap A \cup B)}{P(A \cup B)}$$
$$= \frac{P(B)}{P(A \cup B)}$$
$$= \frac{0.05}{0.14}$$
$$= 0.357$$

(d)
$$P(\bar{A} \mid \bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$$
$$= \frac{1 - P(A \cup B)}{1 - P(B)}$$
$$= \frac{1 - 0.14}{1 - 0.05}$$
$$= 0.905$$

5. (a) A = take the training course

B = meet the production quotas

$P(A) = 0.7$

$P(B|A) = 0.9$

$P(B|\bar{A}) = 0.6$

(b) $P(A \cap B)$

$= P(A) \cdot P(B|A)$ ⇐ since $P(B|A) = \dfrac{P(A \cap B)}{P(A)}$

$= 0.7 \times 0.9$

$= 0.63$

(c) $P(B) = P(B|A) \cdot P(A) + P(B|\bar{A})\, P(\bar{A})$

$= 0.9 \times 0.7 + 0.6 \times 0.3$

$= 0.81$

(d) $P(A|B)$

$= \dfrac{P(A \cap B)}{P(B)}$

$= \dfrac{0.63}{0.81}$ ← from (b) ← from (c)

$= 0.778$

(e) NO, A and B are not independent.

$P(A|B) \neq P(A)$.

$0.778 \neq 0.7$

6. (a) $r_2 = P(A_1 \cup A_2)$

$= 1 - P(\bar{A}_1 \cap \bar{A}_2)$ independent assumption

$= 1 - P(\bar{A}_1) \cdot P(\bar{A}_2)$

$= 1 - (1-p) \cdot (1-p)$

$= 1 - (1-p)^2$

(b) $r_n = P(A_1 \cup A_2 \cup \cdots \cup A_n)$

$= 1 - P(\bar{A}_1 \cap \bar{A}_2 \cap \cdots \cap \bar{A}_n)$ independent assumption

$= 1 - P(\bar{A}_1) \cdot P(\bar{A}_2) \cdots P(\bar{A}_n)$

$= 1 - (1-p)(1-p) \cdots (1-p)$

$= 1 - (1-p)^n$

(c) $r_5 = 1 - (1-p)^5 = 0.9999$

$(1-p)^5 = 0.0001$

$1-p = 0.158$

$p = 0.842$

(d) (A2) Violation example:

Consider a car trip with a spare tire. The reliability (p) of the spare tire may be different from the reliability (p) of the normal tire.

(A1) Violation example:

Consider components of a air-conditioner. Components may fail in many modes such as wear, plastic deformation and instability. In other words, components may fail because of the same variables. Therefore, they are not independent.