Stat 509 Homework 1 Solution

1. (a)

\[ S = \{ \text{LowSBP AB+}, \text{NormalSBP AB+}, \text{HighSBP AB+}, \]
\[ \text{LowSBP AB-}, \text{NormalSBP AB-}, \text{HighSBP AB-}, \]
\[ \text{LowSBP A+}, \text{NormalSBP A+}, \text{HighSBP A+}, \]
\[ \text{LowSBP A-}, \text{NormalSBP A-}, \text{HighSBP A-}, \]
\[ \text{LowSBP B+}, \text{NormalSBP B+}, \text{HighSBP B+}, \]
\[ \text{LowSBP B-}, \text{NormalSBP B-}, \text{HighSBP B-}, \]
\[ \text{LowSBP O+}, \text{NormalSBP O+}, \text{HighSBP O+}, \]
\[ \text{LowSBP O-}, \text{NormalSBP O-}, \text{HighSBP O-} \} \]

\[ n_t = 8 \times 3 = 24 \]

(b)

We assign probabilities to Blood type and Blood pressure level for American male.

<table>
<thead>
<tr>
<th>Blood type</th>
<th>AB+</th>
<th>AB-</th>
<th>A+</th>
<th>A-</th>
<th>B+</th>
<th>B-</th>
<th>O+</th>
<th>O-</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.08</td>
<td>0.005</td>
<td>0.275</td>
<td>0.01</td>
<td>0.31</td>
<td>0.01</td>
<td>0.29</td>
<td>0.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SBP</th>
<th>Low (&lt;90)</th>
<th>Normal (90~140)</th>
<th>High (&gt;140)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.001</td>
<td>0.715</td>
<td>0.284</td>
</tr>
</tbody>
</table>

Assume Blood type and SBP level are independent, then

<table>
<thead>
<tr>
<th>Probability</th>
<th>AB+</th>
<th>AB-</th>
<th>A+</th>
<th>A-</th>
<th>B+</th>
<th>B-</th>
<th>O+</th>
<th>O-</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>0.00008</td>
<td>0.000005</td>
<td>0.000275</td>
<td>0.00001</td>
<td>0.00031</td>
<td>0.00001</td>
<td>0.00029</td>
<td>0.00002</td>
</tr>
<tr>
<td>Normal</td>
<td>0.05720</td>
<td>0.003575</td>
<td>0.196625</td>
<td>0.00715</td>
<td>0.22165</td>
<td>0.00715</td>
<td>0.20735</td>
<td>0.01430</td>
</tr>
<tr>
<td>High</td>
<td>0.02272</td>
<td>0.00142</td>
<td>0.07810</td>
<td>0.00284</td>
<td>0.08804</td>
<td>0.00284</td>
<td>0.08236</td>
<td>0.00568</td>
</tr>
</tbody>
</table>

Sources links:

http://www.bloodpressureuk.org/BloodPressureandyou/Thebasics/Bloodpressurechart

https://obs.health.nokia.com/us/bp

https://pdfs.semanticscholar.org/a929/2b5e6f297fbc591fcf221ab02c29cd05db0b.pdf
(c) \( P(\text{Normal SBP and AB}) \)
\[ = P(\text{Normal SBP and AB}^+) + P(\text{Normal SBP and AB}^-) \]
\[ = 0.0572 + 0.003575 \]
\[ = 0.060775 \]

\[ P(\text{High SBP and O}) \]
\[ = P(\text{High SBP and AB}^+) + P(\text{High SBP and AB}^-) \]
\[ + P(\text{High SBP and A}^+) + P(\text{High SBP and A}^-) \]
\[ + P(\text{High SBP and B}^+) + P(\text{High SBP and B}^-) \]
\[ = 0.02272 + 0.00142 + 0.07810 + 0.00284 + 0.0804 + 0.00284 \]
\[ = 0.19596 \]

\[ P(\text{+ Rh status}) \]
\[ = P(AB^+) + P(A^+) + P(B^+) + P(O^+) \]
\[ = 0.08 + 0.275 + 0.31 + 0.29 \]
\[ = 0.955 \]
2. (a) \[ P(A \cap B) = P(A) \cdot P(B) = 0.2 \]

\[ \Rightarrow \quad P(B) = 0.2 / P(A) = 0.2 / 0.5 = 0.4 \]

\[ P(B) = 1 - P(\overline{B}) = 1 - 0.4 = 0.6 \]

No, we can't calculate \( P(B) \) if \( A \) and \( B \) are not independent.

(b) \[ P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.8 \]

\[ 0.6 + 0.4 - P(A \cap B) = 0.8 \]

\[ P(A \cap B) = 0.2 \]

\[ P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.4} = 0.5 \]

\[ P(B | A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.6} = \frac{1}{3} \]

No, \( A \) and \( B \) are not independent, since \( P(A | B) \neq P(A) \).

(c) \[ P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 \]

\[ \text{plug in } P(A) = 2P(B), \Rightarrow 2P(B) + P(B) - P(A \cap B) = 0.5 \]

\[ \Rightarrow 3P(B) - P(A \cap B) = 0.5 \]

Since \( P(A | B) = P(A) \), \( A \) and \( B \) are independent, \( \Rightarrow 3P(B) - P(A) \cdot P(B) = 0.5 \)

we have \[ P(A \cap B) = P(A) \cdot P(B) \]

\[ 3P(B) - 2P(B) \cdot P(B) = 0.5 \]

\[ P(B) = \frac{3 + \sqrt{5}}{2} \]

\[ \approx 0.382 \] or \( 2.618 \)

since \( 0 \leq P(B) \leq 1 \), we get \( P(B) = 0.382 \).
3. (a) We could assume $A$ and $B$ are independent, since one is in Asia and one in Europe.

However, if those two projects are done using the same technique or managed by the same group of people, $A$ and $B$ would not be independent.

(b) Since we assumed that $A$ and $B$ are independent, $P(\overline{B} | \overline{A}) = P(\overline{B}) = 1 - P(B) = 1 - 0.4 = 0.6$.

(c) $P(\text{at least one will be successful})$

$$= P(A \cap B) + P(A \cap \overline{B}) + P(\overline{A} \cap B)$$

$$= P(A) \cdot P(B) + P(A) \cdot P(\overline{B}) + P(\overline{A}) \cdot P(B)$$

$$= 0.7 \times 0.4 + 0.7 \times 0.6 + 0.3 \times 0.4$$

$$= 0.28 + 0.42 + 0.12$$

$$= 0.82$$

(d) $P(\text{exactly one project will be successful})$

$$= P(A \cap \overline{B}) + P(\overline{A} \cap B)$$

$$= 0.7 \times 0.6 + 0.3 \times 0.4$$

$$= 0.54$$

(e) $P(\text{neither project will be successful})$

$$= P(\overline{A} \cap \overline{B})$$

$$= 0.3 \times 0.6$$

$$= 0.18$$
4. (a) \[ A = \text{catching the usual strain} \]
\[ B = \text{catching the new strain} \]
\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
\[ = 0.1 + 0.05 - 0.01 \]
\[ = 0.14 \]

(b) \[ P(A \mid B) = \frac{P(A \cap B)}{P(B)} \]
\[ = \frac{0.01}{0.05} \]
\[ = 0.2 \]

(c) \[ P(B \mid A \cup B) \]
\[ = \frac{P(B \cap A \cup B)}{P(A \cup B)} \]
\[ = \frac{P(B)}{P(A \cup B)} \]
\[ = \frac{0.05}{0.14} \]
\[ = 0.357 \]

(d) \[ P(\overline{A} \mid B) = \frac{P(\overline{A} \cap B)}{P(B)} \]
\[ = \frac{1 - P(A \cap B)}{1 - P(B)} \]
\[ = \frac{1 - 0.14}{1 - 0.05} \]
\[ = 0.905 \]
5. (a) \( A = \) take the training course
\( B = \) meet the production quotas
\[ P(A) = 0.7 \]
\[ P(B|A) = 0.9 \]
\[ P(B|\overline{A}) = 0.6 \]

(b) \[ P(A \land B) = P(A) \cdot P(B|A) \quad \Leftrightarrow \quad P(B|A) = \frac{P(A \land B)}{P(A)} \]
\[ = 0.7 \times 0.9 \]
\[ = 0.63 \]

(c) \[ P(B) = P(B|A) \cdot P(A) + P(B|\overline{A}) \cdot P(\overline{A}) \]
\[ = 0.9 \times 0.7 + 0.6 \times 0.3 \]
\[ = 0.81 \]

(d) \[ P(A|B) = \frac{P(A \land B)}{P(B)} \quad \text{from (b)} \]
\[ = \frac{0.63}{0.81} \quad \text{from (c)} \]
\[ = 0.778 \]

(e) \( \text{No, } A \text{ and } B \text{ are not independent.} \)
\[ P(A|B) \neq P(A), \]
\[ 0.778 \neq 0.7 \]
6. (a) \[ Y_2 = P(A_1 \cup A_2) \]
\[ = 1 - P(A_1 \cap A_2) \] 
\[ = 1 - P(A_1) \cdot P(A_2) \] 
\[ = 1 - (1-p)(1-p) \] 
\[ = 1 - (1-p)^2 \]

(b) \[ Y_n = P(A_1 \cap A_2 \cap \ldots \cap A_n) \]
\[ = 1 - P(A_1 \cap A_2 \cap \ldots \cap A_n) \] 
\[ = 1 - P(A_1) \cdot P(A_2) \cdots P(A_n) \] 
\[ = 1 - (1-p)(1-p) \ldots (1-p) \] 
\[ = 1 - (1-p)^n \]

(c) \[ Y_5 = 1 - (1-p)^5 = 0.9999 \]
\[ (1-p)^5 = 0.0001 \]
\[ 1 - p = 0.158 \]
\[ p = 0.842 \]

(d) (A2) Violation example:
Consider a car trip with a spare tire. The reliability (p) of the spare tire may be different from the reliability (p) of the normal tire.

(A1) Violation example:
Consider components of an air conditioner. Components may fail in many modes such as wear, plastic deformation and instability. In other words, components may fail because of the same variables. Therefore, they are not independent.