Note: This homework assignment covers Chapter 4.

Disclaimer: If you use R, include all R code and output as attachments. Do not just "write in" the R code you used. Also, don't just write the answer and say this is what R gave you. If my grader can't see how you got an answer, it is wrong. I want to see your code and your answers accompanying your code (like in the notes).

1. UPS ships millions of packages in a specific 1-ft^3 packing container. Let

$$Y = \text{amount of space (in ft}^3\text{) occupied in this container.}$$

The probability density function (pdf) of Y is given by

$$f_Y(y) = \begin{cases} 90y^8(1-y), & 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Graph the pdf of Y. If you want, you can use this R code below:

```
y = seq(0,1,0.01)
pdf = 90*y^8*(1-y)
plot(y,pdf,type="l",xlab="y",ylab="PDF")
abline(h=0) # puts horizontal line at 0
abline(v=1,lty=2) # puts vertical line at 1; "lty" option alters line type
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(b) Find the probability that the contents of a randomly selected 1-ft^3 packing container will occupy less than 1/2 of the volume of the container; i.e., calculate $P(Y < 0.5)$. Shade in the corresponding region on your pdf in part (a) showing this probability.
(c) Find the mean and variance of Y. Indicate the units of each quantity. Place an "$\times$" on the horizontal axis of your pdf in part (a) indicating where $E(Y)$ falls.
(d) I calculated the median of this distribution to be 0.8377 ft^3. Write out an equation (involving an integral) that when solved will give this answer. Make sure you say "what is being solved for" in your equation. You don't have to try to solve the equation.

2. A critical factor in solid missile fuel is the particle size distribution. Significant problems occur if the particle sizes are too large. From production data in the past, it has been determined that the probability density function (pdf) of the particle size (Y, in micrometers) is

$$f_Y(y) = \begin{cases} \dfrac{3}{y^4}, & y > 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Graph the pdf of Y.
(b) Calculate the cumulative distribution function $F_Y(y)$, for $y > 1$. What is $F_Y(y)$ when

$y \leq 1$? Graph $F_Y(y)$.
(c) Find the mean and median of Y. Why is the mean larger?
(d) What is the probability that a random particle from the manufactured fuel exceeds 4 micrometers?

3. The time to failure (Y, measured in hours) of fans in a laptop computer is modeled using an exponential distribution with $\lambda = 0.0002$.

(a) Graph the pdf of Y. Compute $E(Y)$ and var(Y). Place an "$\times$" on the pdf indicating where $E(Y)$ is.
(b) What is the probability that a fan will fail before 6,000 hours? will survive at least 12,000 hours?
(c) Only 1 percent of all fans' lifetimes will exceed which value?
(d) In a class of 25 students, each student has been provided with his/her own new laptop with this type of fan. Let X denote the number of students (out of 25) whose fan will fail before 6,000 hours. What is the probability that 3 or fewer students' fans will fail before 6,000 hours; i.e., $P(X \leq 3)$? State the assumptions you are making about the 25 computers for this calculation to be applicable.

4. The time to death (Y, measured in days) for patients with a serious type of advanced tongue cancer follows a gamma distribution with $\alpha = 2.7$ and $\lambda = 0.01$.

(a) Graph the pdf of Y. Compute $E(Y)$ and var(Y). Place an "$\times$" on the pdf indicating where $E(Y)$ is.
(b) What is the probability a patient with this type of cancer will live longer than one year? Note that 1 year is 365 days.
(c) Find the 80th percentile of this distribution and interpret what it means.

5. In non-catastrophe settings, administrators have determined that patients arrive at the Richland Hospital emergency room according to a Poisson process with mean $\lambda = 3.5$ per hour.

(a) What is the probability the ER staff will have to wait longer than 30 minutes to receive the first patient? Note that 30 minutes is 0.5 hours.
(b) Let X denote the time until the 3rd patient is received. Give the distribution of X and find $P(X \leq 0.25)$; i.e., the probability that the 3rd patient is received in 15 minutes or less.
(c) Why might it be useful to have knowledge of wait time probabilities like those in parts (a) and (b)?

6. The demand for daily water use in Phoenix during the summer is a random variable Y, measured in millions of gallons. Suppose the distribution of Y is normal (Gaussian)

with mean $\mu = 310$ and standard deviation $\sigma = 45$.

(a) Graph the pdf of Y. Place an "$\times$" on the pdf indicating where $E(Y) = \mu$ is.
(b) City reservoirs have a combined storage capacity of 400 million gallons per day. What is the probability a day requires more water than is stored in city reservoirs?
(c) What reservoir capacity is needed so that the probability the demand exceeds the capacity is only 0.05?