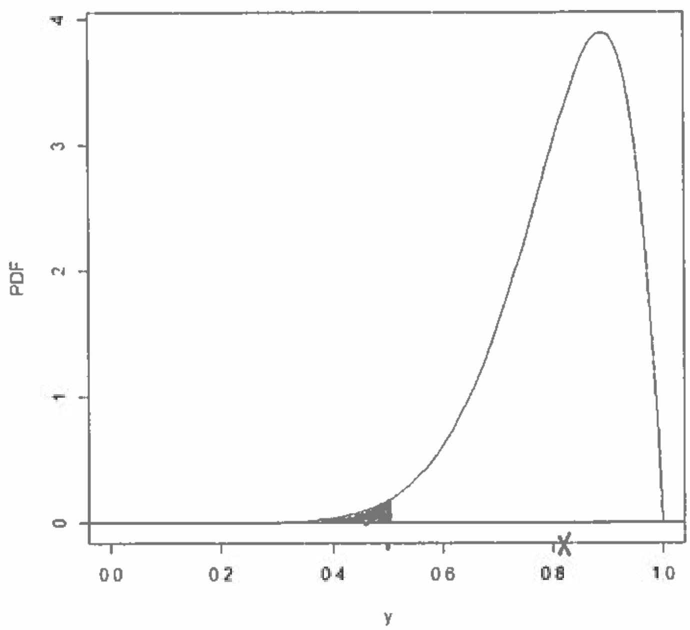
# Stat 509 Homework 3 Solution

1. (a)



(b) $P(Y < 0.5)$

$$= \int_0^{0.5} 90y^8(1-y)\,dy$$

$$= \int_0^{0.5} 90y^8\,dy - \int_0^{0.5} 90y^9\,dy$$

$$= \frac{90}{9} y^9 \Big|_0^{0.5} - \frac{90}{10} y^{10} \Big|_0^{0.5}$$

$$= 10 \times 0.5^9 - 9 \times 0.5^{10}$$

$$= 0.0107$$

(c) $E(Y) = \int_0^1 y\, f_Y(y)\,dy$

$$= \int_0^1 y \cdot 90y^8(1-y)\,dy$$

$$= \int_0^1 90y^9\,dy - \int_0^1 90y^{10}\,dy$$

$$= \frac{90}{10} y^{10} \Big|_0^1 - \frac{90}{11} y^{11} \Big|_0^1$$

$$= 9 - \frac{90}{11}$$

$$= 0.8182$$

$E(Y^2) = \int_0^1 y^2 f_Y(y)\,dy$

$$= \int_0^1 y^2 \cdot 90y^8(1-y)\,dy$$

$$= \int_0^1 90y^{10}\,dy - \int_0^1 90y^{11}\,dy$$

$$= \frac{90}{11} y^{11} \Big|_0^1 - \frac{90}{12} y^{12} \Big|_0^1$$

$$= \frac{90}{11} - \frac{90}{12}$$

$$= 0.6818$$

$Var(Y) = E(Y^2) - [E(Y)]^2$

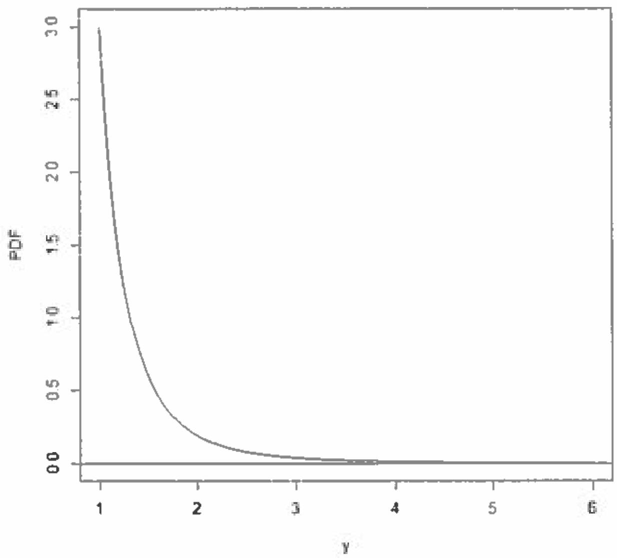
$$= 0.6818 - 0.8182^2$$

$$= 0.0123$$

(d) $\int_0^y 90t^8(1-t)\,dt = 0.5$

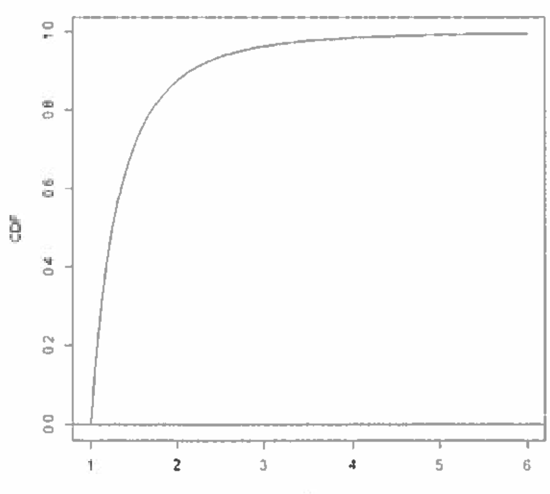
Solve this equation, we will get the median $y = 0.8377$.

2. (a)




```
y = seq(1,6,0.01)
pdf = 3/y^4
plot(y,pdf,type="l",xlab="y",ylab="PDF")
abline(h=0) # puts horizontal line at 0
```

(b)




```
y = seq(1,6,0.01)
cdf = 1-y^(-3)
plot(y,cdf,type="l",xlab="y",ylab="CDF")
abline(h=0) # puts horizontal line at 0
```

(b) when $y>1$

$$F_Y(y) = \int_1^y \frac{3}{t^4}\, dt$$

$$= \frac{3}{-3} t^{-3} \Big|_1^y$$

$$= 1 - y^{-3}$$

when $y \le 1$

$$F_Y(y) = 0$$

(c) $$E(Y) = \int_1^\infty y \cdot \frac{3}{y^4}\, dy$$

$$= \int_1^\infty 3 \cdot y^{-3}\, dy$$

$$= -\frac{3}{2} y^{-2} \Big|_1^\infty$$

$$= \frac{3}{2}$$

$$\int_1^x \frac{3}{y^4}\, dy = 0.5$$

$$\frac{3}{-3} y^{-3} \Big|_1^x = 0.5$$

$$-x^{-3} + 1 = 0.5$$

$$x^{-3} = 0.5$$

$x = \sqrt[3]{2} = 1.2599$ ← median

mean is larger because the density function is skew to the right.

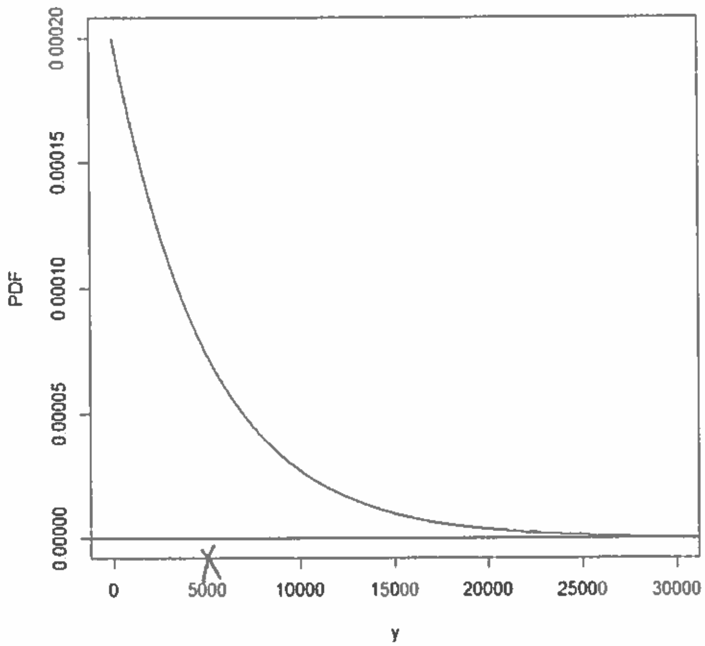
(d) $P(Y > 4)$

$$= \int_4^\infty \frac{3}{t^4}\, dt$$

$$= \frac{3}{-3} t^{-3} \Big|_4^\infty$$

$$= 4^{-3} = 0.0156$$

3. (a)




```
y = seq(0,30000,0.1)
pdf=dexp(y,0.0002)
plot(y,pdf,type="l",xlab="y",ylab="PDF")
abline(h=0) # puts horizontal line at 0
```

$Y \sim$ exponential $(0.0002)$

$E(Y) = \lambda^{-1} = \frac{1}{0.0002} = 5000$

$var(Y) = \lambda^{-2} = \frac{1}{0.0002^2} = 25{,}000{,}000$

(b) $P(Y < 6000)$ 

$= F_Y(6000)$

$= 1 - e^{-0.0002 \times 6000}$

$= 0.6988$

R-code

```
> pexp (6000, 0.0002 )
```

$P(Y > 12,000)$

$= 1 - P(Y \leq 12,000)$

$= 1 - F_Y(12,000)$

$= 1 - (1 - e^{-0.0002 \times 12000})$

$= 0.0907$

R-code

```
> 1 - pexp (12 000 , 0.0002)
```

(c) $P(Y > y) = 0.01 \Rightarrow P(Y \leq y) = 0.99$

$\cancel{1 - P(Y > y)}$ $F_Y(y) = 0.99$

$1 - e^{-0.0002y} = 0.99$

$e^{-0.0002y} = 0.01$

$y = 23025.85$

R-code

```
> q exp (0.99, 0.0002)
```

(d) From (b) we have $P(Y < 6000) = 0.6988$

$X \sim b(25, 0.6988)$

$P(X \leq 3)$

$= 2.8457 \times 10^{-9}$

R-code

```
> pbinom (3, 25, 0.698
```

① Each computer results in fan failure ("success") or fan not failure ("failu

② Computers are independent.

③ The probability of fan failure is the same on every computers.

R-code

(b)

```
> pexp(6000,0.0002)
[1] 0.6988058
```

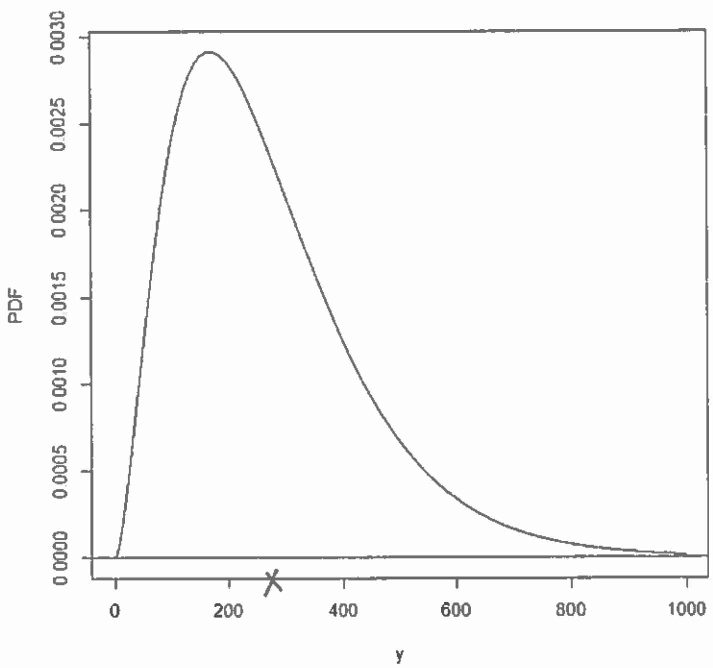
(c)

```
> 1-pexp(12000,0.0002)
[1] 0.09071795
```

(d)

```
> pbinom(3,25,0.6988)
[1] 2.845734e-09
```

4. (a)




```
y = seq(0,1000,0.01)
pdf = dgamma(y,2.7,0.01)
plot(y,pdf,type="l",xlab="y",ylab="PDF")
abline(h=0) # puts horizontal line at 0
```

$Y \sim Gamma(2.7,\ 0.01)$

$E(Y) = \frac{\alpha}{\lambda} = \frac{2.7}{0.01} = 270$

$var(Y) = \frac{\alpha}{\lambda^2} = \frac{2.7}{0.01^2} = 27000$

(b) $P(Y > 365)$ R-code : >1 - pgamma(365, 2.7, 0.01)

$= 1 - P(Y \leq 365)$

$= 1 - F_Y(365)$

$= 0.2355$

(c) $P(Y < y) = 0.8$ R-code : >qgamma(0.8, 2.7, 0.01)

$y = 390.003$

80% of the patients live less than 390 days.

(b)

```
> 1-pgamma(365,2.7,0.01)
[1] 0.2355346
```

(c)

```
> qgamma(0.8,2.7,0.01)
[1] 390.003
```

5.(a) Let $Y$ denote the time until the first patient is received. Then $Y \sim$ exponential $(3.5)$

$P(Y > 0.5)$

$= 1 - P(Y \leq 0.5)$ R-code : > 1- pexp(0.5, 3.5)

$= 1 - F_Y(0.5)$

$= 0.1738$

(b) $X \sim$ Gamma $(3, 2.5)$

R-code : > pgamma(0.25, 3, 2.5)

$P(X \leq 0.25)$

$= F_X(0.25)$

$= 0.0257$

(c) The knowledge of waiting time probability helps administrator to allocate the right amount of staff to emergence room.

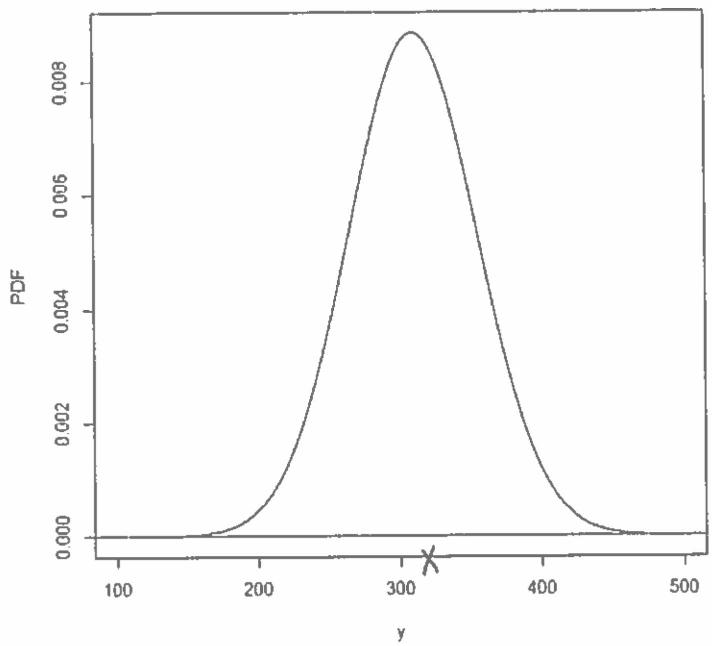
(a)

```
> 1-pexp(0.5,3.5)
[1] 0.1737739
```

(b)

```
> pgamma(0.25,3,2.5)
[1] 0.02565693
```

6.(a)




```
y = seq(100,500,0.01)
pdf = dnorm(y,310,45)
plot(y,pdf,type="l",xlab="y",ylab="PDF")
abline(h=0) # puts horizontal line at 0
```

(b) $P(Y > 400)$

$= 1 - P(Y \leq 400)$

$= 0.02275$

```
> 1-pnorm(400,310,45)
[1] 0.02275013
```

(c) $P(Y > y) = 0.05$

$P(Y \leq y) = 1 - 0.05 = 0.95$

$y = 384.0184$

```
> qnorm(0.95,310,45)
[1] 384.0184
```