STAT 509 HW4 Solution

1. (a) See graph.

$E(T) = \eta\,\Gamma(1+\frac{1}{\beta}) = 17.55\,\Gamma(1+\frac{1}{1.39}) = 16.01372$

(b) $F_T(\phi_{0.5}) = 1 - e^{-(\phi_{0.5}/17.55)^{1.39}} = 0.5$

$\phi_{0.5} = 13.48227$

Half of the dialysis patients' survival time is below 13.5 months.

(c) $S_T(t) = 1 - F_T(t) = e^{-(t/17.55)^{1.39}}$ See graph.

$S_T(24) = e^{-(24/17.55)^{1.39}} = 0.2132967$

The probability that a dialysis patient will survive beyond 24 months is 0.21.

(d) $F_T(\phi_{0.95}) = 1 - e^{-(\phi_{0.95}/17.55)^{1.39}} = 0.95$

$\phi_{0.95} = 38.64426$

95% of the patients will die before 38.64 months.

(e) $\beta = 1.39 > 1 \Rightarrow h_T(t)$ is increasing.

The population of dialysis patients with chronic kidney disease gets weaker over time.

2. (a) See R-code

$\hat{\beta} = 3.873157$

$\hat{\eta} = 6.920191$

(b) $P(T < 5.0) = F_T(5.0) = 0.2472299$

24.7% of the turbo chargers will fail before 5,000 hours.

(c) $F_T(\phi_{0.9}) = 1 - e^{-(\phi_{0.9}/6.920191)^{3.873157}} = 0.9$

$\phi_{0.9} = 8.58297$ (1000s of hours)

90% of the turbochargers will fail before 8582.97 hours.

(d) See graph

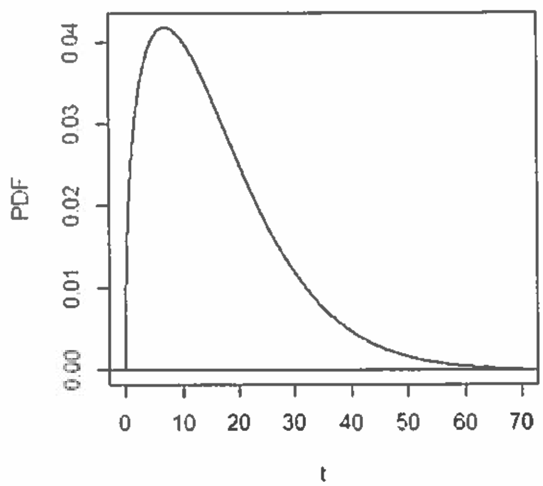
Hazard function is increasing. The population of turbo chargers gets weaker over time.

(e) See graph.

In general, weibull model seems reasonable for these data. The qq plot has a little curvature on the tails, however, since it's within the bands, we will not suspect the model.

The answers to parts (b) (c) and (d) are ok.

1. (a)

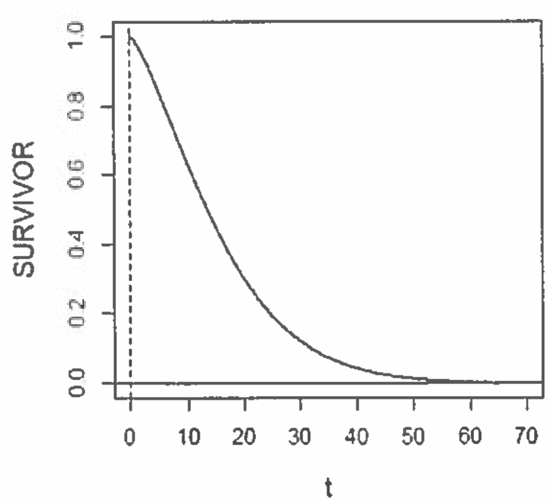



```
> t = seq(0,70,0.01)
> plot(t,dweibull(t,1.39,17.55),type="l",lty=1,xlab="t",ylab="PDF")
> abline(h=0)
> 17.55*gamma(1+1/1.39)
[1] 16.01372
```

(b)

```
> qweibull(0.5,1.39,17.55)
[1] 13.48227
```

(c)




```
> survivor = 1-pweibull(t,1.39,17.55)
> plot(t,survivor,type="l",xlab="t",ylab="SURVIVOR",ylim=c(0,1),cex.lab=1.25)
```

```
> abline(h=0)
> abline(v=0,lty=2)
> 1-pweibull(24,1.39,17.55)
[1] 0.2132967
```

(d)

```
> qweibull(0.95,1.39,17.55)
[1] 38.64426
```

2. (a)

```
> turbodata=c(1.6, 2.0, 2.6, 3.0, 3.5, 3.9, 4.5, 4.6, 4.8, 5.0,
+      5.1, 5.3, 5.4, 5.6, 5.8, 6.0, 6.0, 6.1, 6.3, 6.5,
+      6.5, 6.7, 7.0, 7.1, 7.3, 7.3, 7.3, 7.7, 7.7, 7.8,
+      7.9, 8.0, 8.1, 8.3, 8.4, 8.4, 8.5, 8.7, 8.8, 9.0)
> library(fitdistrplus)
> fitdist(turbodata,"weibull")
Fitting of the distribution ' weibull ' by maximum likelihood
Parameters:
      estimate Std. Error
shape 3.873157  0.5176799
scale 6.920191  0.2946851
```

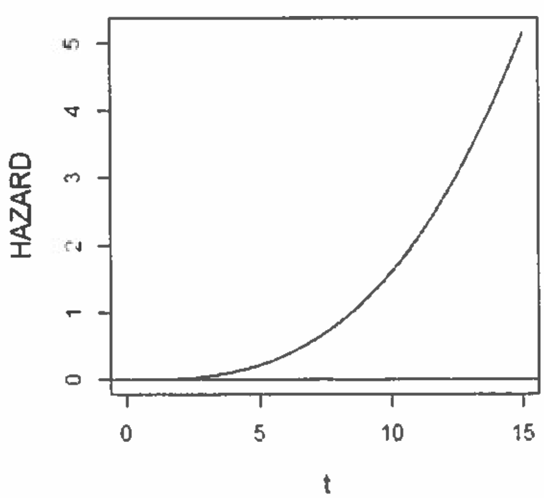
(b)

```
> pweibull(5,3.873157,6.920191)
[1] 0.2472299
```

(c)

```
> qweibull(0.9,3.873157,6.920191)
[1] 8.58297
```

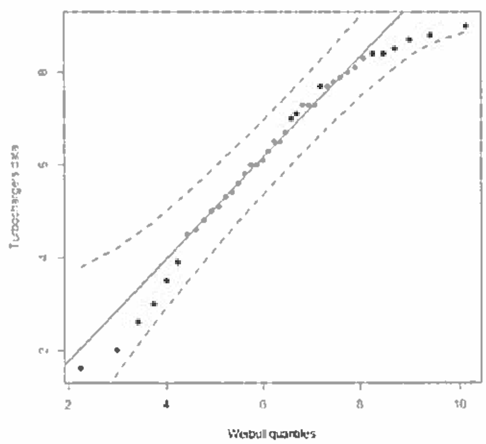
(d)




```
> t=seq(0,15,0.01)
> pdf = dweibull(t,3.873157,6.920191)
```

```
> survivor = 1-pweibull(t,3.873157,6.920191)
> hazard = pdf/survivor
> plot(t,hazard,type="l",xlab="t",ylab="HAZARD",cex.lab=1.25)
> abline(h=0)
```

(e)




```
> library(car)
> qqPlot(turbodata,distribution="weibull",shape=beta.hat,scale=eta.hat,xlab="
Weibull quantiles",ylab="Turbochargers data",pch=16)
```

3. (a) see graph.

(b) $E(T) = e^{\mu + \sigma^2/2} = e^{10 + 2.25/2} = 67846.29$

$F_T(\phi_{0.5}) = 0.5 \quad \phi_{0.5} = 22026.47$

The Mean is larger than the median. (skew to the right).

(c) $P(T > 100{,}000) = 1 - F_T(100{,}000) = 0.1565792$

(d) $F_T(\phi_{0.1}) = 0.1 \quad \phi_{0.1} = 3221.726$

10% of the semiconductor lasers will fail before 3221.726 hours.

(e) See the hazard function graph.

The hazard function of lognormal distribution rises to a peak quickly, then decrease overtime. ~~It is not~~

Compare with lognormal distribution, weibull distribution's hazard function is more flexiable. eg. $h_T(t)$ is increasing if $\beta > 1$; $h_T(t)$ is constant is $\beta = 1$; $h_T(t)$ is decreasing if $\beta < 1$.

(f) If weibull is used to fit the data, the estimated hazard function will not capture the increasing pattern starts from 0, which is the pattern that a lognormal hazard function should have. It means that at the begining of lifetime, the estimation is not reliable.

4\. (a) $\hat{\beta} = 1.370963$

$\hat{\eta} = 131.340111$

(b) $\phi_{0.5} = 100.5294$

Weibull model may not be a reasonable model for the data.

(c) ① See graph.

qq plot shows curvature in it. Weibull model is not a reasonable model.

② $\hat{\mu} = 4.4992858$

$\hat{\sigma} = 0.7800686$

③ $\phi_{0.5} = 89.95286$

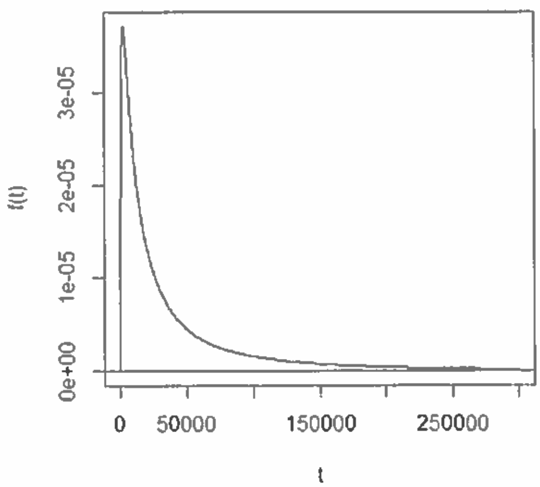
No, they are not close.

④ see graph.

Yes, lognormal model is a better fit to these data, because qq plot looks like a straight line.

(d) see graph.

3. (a)



(b)

```
> qlnorm(0.5,10,sqrt(2.25))
[1] 22026.47
```

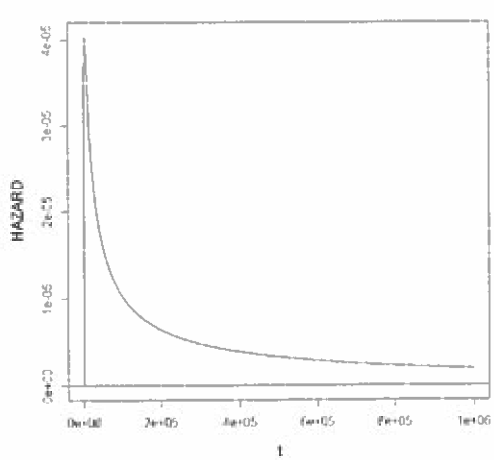
(c)

```
> 1-plnorm(100000,10,sqrt(2.25))
[1] 0.1565792
```

(d)

```
> qlnorm(0.1,10,sqrt(2.25))
[1] 3221.726
```

(e)



```
> t = seq(0,1000000,1)
>  pdf = dlnorm(t,10,sqrt(2.25))
> survivor = 1-plnorm(t,10,sqrt(2.25))
> hazard = pdf/survivor
> plot(t,hazard,type="l",xlab="t",ylab="HAZARD",cex.lab=1.25)
>  abline(h=0)
```

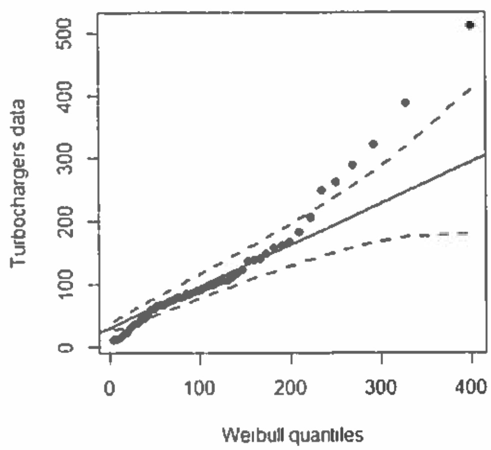
4. (a)

```
 drill_data=c(11, 14, 20, 23 ,31, 36, 39, 44, 47, 50,
 59, 61, 65, 67, 68, 71, 74, 76, 78, 79,
 81 ,84, 85, 89, 91, 93, 96, 99 ,101 ,104,
 105 ,105, 112, 118, 123, 136, 139, 141, 148, 158,
 161, 168, 184, 206, 248, 263, 289, 322, 388, 512)
> library(fitdistrplus)
Loading required package: MASS
Loading required package: survival
> fitdist(drill_data,"weibull")
Fitting of the distribution ' weibull ' by maximum likelihood
Parameters:
        estimate Std. Error
shape   1.370963  0.1409621
scale 131.340111 14.3520854
```

(b)

```
> qweibull(0.5, 1.370963,131.340111)
[1] 100.5294
```

(c) 1.




```
> library(car)
> qqPlot(drill_data,distribution="weibull",shape= 1.370963,scale=131.340111,x
lab="Weibull quantiles",ylab="Turbochargers data",pch=16)
```

(c) 2.

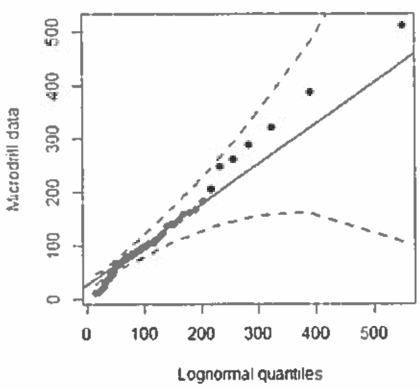
```
> fitdist(drill_data,"lnorm")
Fitting of the distribution ' lnorm ' by maximum likelihood
Parameters:
        estimate Std. Error
```

```
meanlog 4.4992858 0.11031837
sdlog   0.7800686 0.07800629
```

(c) 3.

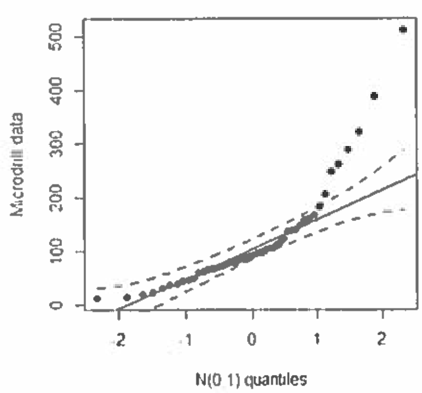
```
> qlnorm(0.5,4.4992858,0.7800686)
[1] 89.95286
```

(c) 4.




```
> meanlog = 4.499
> sdlog = 0.780
> qqPlot(drill_data,distribution="lnorm",meanlog=meanlog,sdlog=sdlog,
+        xlab="Lognormal quantiles",ylab="Microdrill data",pch=16)
```

(d)




```
> qqPlot(drill_data,distribution="norm",xlab="N(0,1) quantiles",
+        ylab="Microdrill data",pch=16)
```