GROUND RULES:

- Print your name at the top of this page.
- This is a closed-book and closed-notes exam. A list of discrete and continuous distributions appears at the end.
- You may use a calculator. **Translation:** Show all of your work; use a calculator only to do final calculations and/or to check your work.
- This exam contains 6 questions. Each question is worth 10 points. This exam is worth 60 points.
- Each question contains subparts. On each part, there is opportunity for partial credit, so show all of your work and explain all of your reasoning. **Translation:** No work/no explanation means no credit.
- Any discussion or inappropriate communication between you and another examinee, as well as the appearance of any unnecessary material, will result in a very bad outcome for you.
- You have 75 minutes to complete this exam.

HONOR PLEDGE FOR THIS EXAM:

After you have finished the exam, please read the following statement and sign your name below it.

> *I promise that I did not discuss any aspect of this exam with anyone other than the instructor, that I neither gave nor received any unauthorized assistance on this exam, and that the work presented herein is entirely my own.*

1. In a recent report from the US Health Resources and Services Administration, the following population-level characteristics were presented:

- 1 percent of Americans were infected with HCV
- among those Americans infected with HCV, 3 percent were infected with HIV
- among those Americans infected with HIV, 10 percent were infected with HCV.

(a) Define two relevant events using set notation and interpret the three percentages above in terms of probabilities (two are conditional probabilities).

(b) Find the probability that an individual has HIV.

(c) Are the two viral diseases independent? Prove your answer.

2. A web host has 4 independent servers connected in parallel. At least 3 of them must be operational for the web service to be operational.

(a) If individual servers are operational with probability 0.95, what is probability the web service is operational? Answer this question by using the binomial distribution; e.g., let Y denote the number of operational servers.

(b) Suppose the individual servers have different probabilities of being operational: 0.70, 0.80, 0.99, and 0.99, respectively. What is the probability the web service is operational now? Why can't you use the binomial distribution to answer this question?

3. A blood bank receives donors in succession. Here is the distribution of the blood types in the US population:

O^+	O^-	A^+	A^-	B^+	B^-	AB^+	AB^-
0.38	0.07	0.34	0.06	0.09	0.02	0.03	0.01

Assume this distribution is accurate when answering the questions below. Treat each donor as independent.

(a) What is the probability the first O^+ blood type donor will be seen among the first 4 donors who visit the bank?

(b) The blood bank will remain open until it receives 5 donors who are AB^+ or AB^-. What probability distribution describes the number of donors that will be seen in total? Be specific.

(c) In part (b), what is the mean number of donors that will be seen in total? *Note:* If you cannot remember the correct formula, use your intuition (and explain yourself).

4. Automobiles arrive at a vehicle equipment inspection station according to a Poisson process with mean $\lambda = 4$ per hour.

(a) What is the probability no more than 2 vehicles will arrive in a given hour?

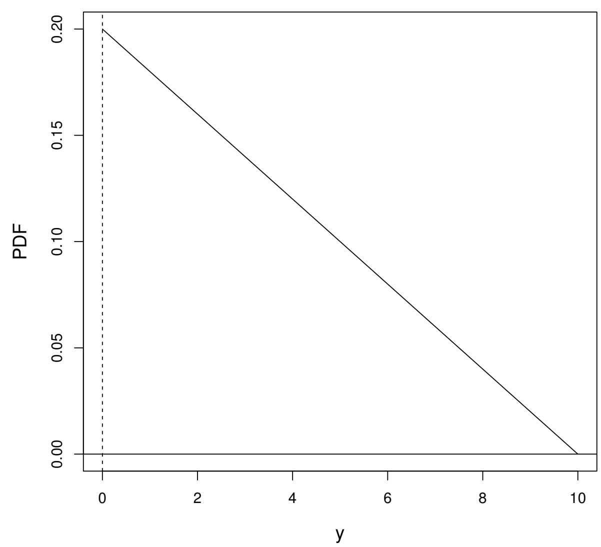
(b) Let T denote the time until the first vehicle arrives (in hours). Find the probability the station will have to wait at least 30 minutes for the first vehicle to arrive. Note that 30 minutes is 1/2 of an hour.

(c) What distribution describes the time until the 4th vehicle arrives? Give me the name of the distribution and the values of the parameters in it.

5. The amount of gravel (in tons) sold by a construction company on a given day is modeled as a continuous random variable Y with probability density function (pdf):

$$f_Y(y) = \begin{cases} 0.02(10 - y), & 0 < y < 10 \\ 0, & \text{otherwise.} \end{cases}$$

A graph of this pdf is shown below:



(a) What is the probability the company will sell less than 5 tons on a given day?

(b) Calculate $E(Y)$. <u>Interpret in words</u>. Use the back of this page if you need more space.

6. Resistors used in the construction of an aircraft guidance system have lifetimes T (in 100s of hours) that are modeled using a Weibull distribution. From years of historical data, engineers use $\beta = 1.5$ and $\eta = 150$.

(a) Calculate the probability a randomly selected resistor will have a lifetime between 10,000 and 20,000 hours. That is, calculate $P(100 < T < 200)$.

(b) Ninety percent of resistors will fail before what time?

(c) Recall that the hazard function of T is

$$h_T(t) = \frac{f_T(t)}{S_T(t)},$$

where $S_T(t) = 1 - F_T(t)$ is the survivor function. Graph the hazard function in this example and interpret what it means. Use the back of this page if you need more space.

Binomial:

$$p_Y(y) = \begin{cases} \binom{n}{y} p^y (1-p)^{n-y}, & y = 0, 1, 2, ..., n \\ 0, & \text{otherwise.} \end{cases}$$

Geometric:

$$p_Y(y) = \begin{cases} (1-p)^{y-1} p, & y = 1, 2, 3, ... \\ 0, & \text{otherwise.} \end{cases}$$

Negative binomial:

$$p_Y(y) = \begin{cases} \binom{y-1}{r-1} p^r (1-p)^{y-r}, & y = r, r+1, r+2, ... \\ 0, & \text{otherwise.} \end{cases}$$

Hypergeometric:

$$p_Y(y) = \begin{cases} \dfrac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}, & y \le r \text{ and } n - y \le N - r \\ 0, & \text{otherwise.} \end{cases}$$

Poisson:

$$p_Y(y) = \begin{cases} \dfrac{\lambda^y e^{-\lambda}}{y!}, & y = 0, 1, 2, ... \\ 0, & \text{otherwise.} \end{cases}$$

Exponential:

$$f_Y(y) = \begin{cases} \lambda e^{-\lambda y}, & y > 0 \\ 0, & \text{otherwise.} \end{cases} \qquad F_Y(y) = \begin{cases} 1 - e^{-\lambda y}, & y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Gamma:

$$f_Y(y) = \begin{cases} \dfrac{\lambda^\alpha}{\Gamma(\alpha)} y^{\alpha-1} e^{-\lambda y}, & y > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Normal (Gaussian):

$$f_Y(y) = \begin{cases} \dfrac{1}{\sqrt{2\pi}\sigma} e^{-(y-\mu)^2/2\sigma^2}, & -\infty < y < \infty \\ 0, & \text{otherwise.} \end{cases}$$

Weibull:

$$f_T(t) = \begin{cases} \dfrac{\beta}{\eta}\left(\dfrac{t}{\eta}\right)^{\beta-1} e^{-(t/\eta)^\beta}, & t > 0 \\ 0, & \text{otherwise.} \end{cases} \qquad F_T(t) = \begin{cases} 1 - e^{-(t/\eta)^\beta}, & t > 0 \\ 0, & \text{otherwise.} \end{cases}$$