GROUND RULES:

- Print your name at the top of this page.
- This is a closed-book and closed-notes exam.
- You may use a calculator. **Translation:** Show all of your work; use a calculator only to do final calculations and/or to check your work.
- This exam contains 4 questions. Each question is worth 15 points. This exam is worth 60 points.
- Each question contains subparts. On each part, there is opportunity for partial credit, so show all of your work and explain all of your reasoning. **Translation:** No work/no explanation means no credit.
- Any discussion or inappropriate communication between you and another examinee, as well as the appearance of any unnecessary material, will result in a very bad outcome for you.
- You have 75 minutes to complete this exam.

HONOR PLEDGE FOR THIS EXAM:

After you have finished the exam, please read the following statement and sign your name below it.

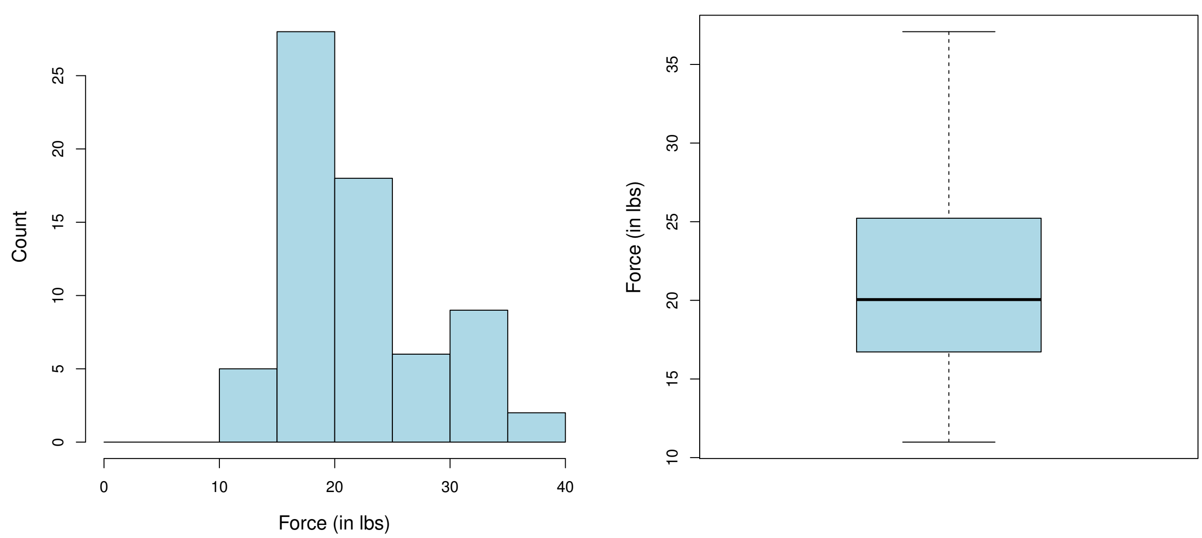
> *I promise that I did not discuss any aspect of this exam with anyone other than the instructor, that I neither gave nor received any unauthorized assistance on this exam, and that the work presented herein is entirely my own.*

1. The force needed to remove a cap from a medicine bottle is an important characteristic. The force must be sufficiently high to prevent unauthorized entry (e.g., by children, etc.) but not so large that elderly patients cannot open the bottle.

A random sample of $n = 68$ bottles was taken from a production process. The force (in lbs) was measured on each bottle by an automated device.

14.7	18.9	27.6	24.3	24.1	28.7	22.2	21.9	16.5	17.6
22.9	16.0	16.1	18.4	30.4	16.2	14.5	15.3	25.7	15.3
16.1	15.2	15.2	19.8	19.1	10.5	22.1	17.1	15.6	17.5
20.2	17.5	20.3	15.9	17.7	20.7	24.2	27.6	17.3	33.0
31.9	27.2	21.1	21.3	26.0	31.2	34.3	32.5	24.9	16.9
37.1	36.1	34.4	20.2	20.0	21.8	14.0	15.0	19.6	15.7
30.4	24.3	15.6	17.2	17.8	21.1	34.9	24.9		

Here is a histogram and a boxplot of the data:



I used R to calculate the sample mean and the sample standard deviation of the data above:

```
> mean(force)
[1] 21.7
> sd(force)
[1] 6.4
```

Four questions appear on the next two pages.

(a) What do the sample mean and the sample standard deviation estimate? Explain in words—don't just write symbols.

(b) Recall the Central Limit Theorem, which says the sample mean

$$\overline{Y} \sim \mathcal{AN}\left(\mu, \frac{\sigma^2}{n}\right),$$

where μ is the population mean and σ^2 is the population variance. Using this result, calculate an estimate of the standard error of the sample mean. Describe in words what this measures.

Two additional questions appear on the next page.

(c) I used R to calculate a 99 percent confidence interval for the population mean force required:

```
> t.test(force,conf.level=0.99)$conf.int
[1] 19.7 23.9
```

Interpret what this confidence interval means.

(d) One engineer examines the interval and notes that only 14 out of the 68 force measurements are between 19.7 and 23.9. She concludes,

> *"14 out of 68 is a lot less than 99 percent. Therefore, the statistical assumptions must be violated."*

Do you agree or disagree with this statement? Explain. *Note:* Do not waste time verifying the 14/68 statement.

2. A geneticist is interested in estimating the proportion of males in South Carolina (aged 18 or older) who have a certain minor blood disorder. From a small study involving 200 male subjects (all South Carolinians), she determines that 36 have the disorder.

(a) What is the population here? What is the sample?

(b) Calculate a 90 percent confidence interval for the population proportion using

$$\widehat{p} \pm z_{\alpha/2}\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}}$$

Interpret your interval. Note that for 90 percent confidence the quantile $z_{0.10/2} \approx 1.65$.

One additional question appears on the next page.

(c) The geneticist wants to use the results above to design a larger study. She would like to calculate a 99 percent confidence interval ($z_{0.01/2} \approx 2.58$) for the population proportion that has margin of error equal to 0.01.

How many males will she have to sample? Comment on whether you think this is feasible and then discuss ways she could reduce the number of males needed.

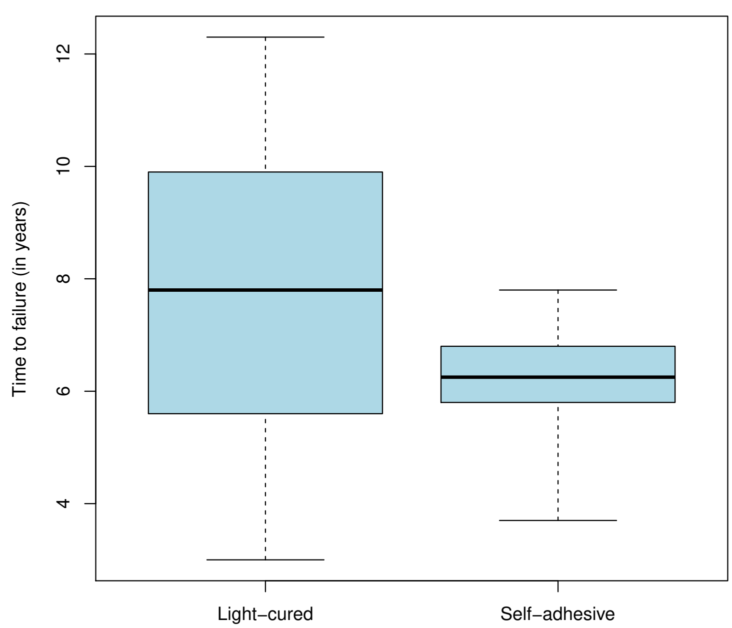
3. Dental veneers are wafer-thin shells designed to cover the front surface of teeth. An orthodontist would like to compare the population mean lifetimes of veneers that differ only in what type of resin cement was used to attach them. Two types of cement were included: (1) light-cured resin cement and (2) self-adhesive resin cement.

The orthodontist sampled 14 patients who received a veneer applied to a top front tooth using light-cured resin cement. An independent sample was taken for 14 patients whose front-tooth veneer was applied using self-adhesive resin cement. The times to failure (in years) were recorded for each patient; these data are below.

For example, the first patient's veneer in the light-cured resin cement group lasted for 7.6 years before it failed (e.g., it fell off, cracked, chipped, etc.).

Light-cured		Self-adhesive	
7.6	3.0	5.4	3.7
12.3	9.9	6.2	7.8
5.6	9.6	6.3	6.0
6.7	8.6	6.5	5.0
5.6	10.7	6.0	6.8
4.4	10.0	7.0	6.6
8.0	7.3	6.8	5.8

Here are side-by-side boxplots of the data:



Three questions appear on the next two pages.

(a) If you wanted to write a confidence interval for $\mu_1 - \mu_2$, the difference of the two population mean failure times between resin cement types (1 = light-cured; 2 = self-adhesive), select the confidence interval you would use:

- the one that assumed equal population variances $\sigma_1^2 = \sigma_2^2 = \sigma^2$.
- the one that did not assume the population variances were equal.

Explain why you chose the answer you did. In addition, what statistical procedure could be used to determine which assumption is more reasonable? (explanation only; no calculations are needed here).

(b) I asked R to get both intervals (each at the 95 percent confidence level). Here are the intervals:

```
> t.test(light.cured,self.adhesive,conf.level=0.95,var.equal=TRUE)$conf.int
[1] 0.14 3.19
> t.test(light.cured,self.adhesive,conf.level=0.95,var.equal=FALSE)$conf.int
[1] 0.10 3.24
```

For the interval that you picked in part (a), interpret the corresponding interval here.

One additional question appears on the next page.

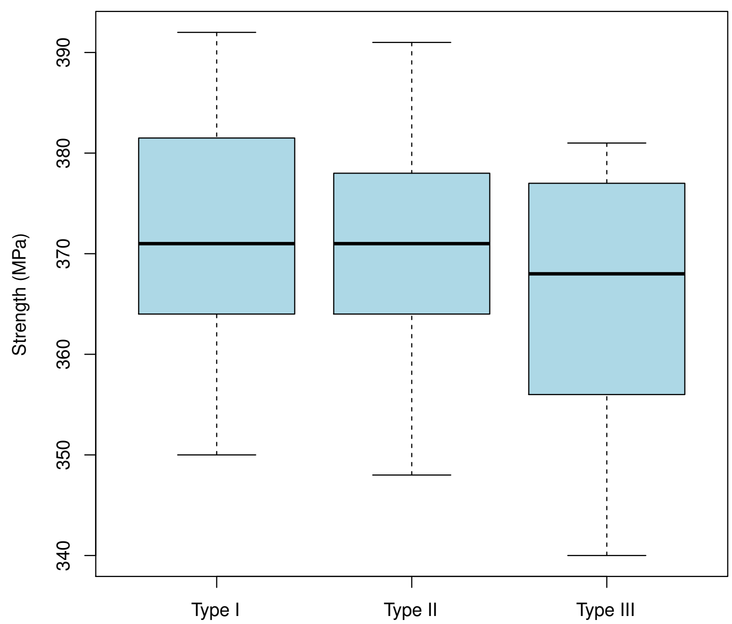
(c) The data above are from an observational study. The orthodontist simply selected the samples of patients from dental records. Could the orthodontist perform an experiment using a matched-pairs design to compare the population mean resin cement failure times? Explain how she could do this with 14 new patients and why this design might be better than using two independent samples.

4. Wire rope is a device that helps to support and move an object or load. In the lifting and rigging industries, wire rope is attached to a crane or hoist and fitted with swivels, shackles, or hooks to attach to a load and move it in a controlled matter. It can also be used to lift and lower elevators or as a means of support for suspension bridges or towers.

Engineers performed an observational study to compare the population mean strengths of three different types of wire rope. Fifteen specimens of each type were selected, and the strength (in MPa) was determined for each specimen. Here are the data:

Type I:	350	351	352	358	370	370	371	371	372	375	379	384	391	391	392
Type II:	348	354	359	363	365	368	369	371	373	374	376	380	383	388	391
Type III:	340	343	352	354	358	359	361	368	372	373	375	379	380	380	381

Here are side-by-side boxplots of the data:



(a) In performing an analysis of variance, what two hypotheses does the overall F statistic test? You can write your answer out in words, or you can use statistical symbols. If you use symbols, define what the symbols mean.

Three additional questions appear on the next page.

(b) Here is the R output from the analysis of variance:

```
Response: strength
          Df Sum Sq Mean Sq F value Pr(>F)
type       2  404.4  202.20  1.1316 0.3322
Residuals 42 7504.8  178.69
```

Note that the overall F statistic is $F \approx 1.13$ and the probability value is ≈ 0.33. Using the hypotheses you described in part (a), make a decision as to which hypothesis is more supported by the data. Explain your decision.

(c) Although the overall F test is robust to normality departures, it is not robust to a violation in the equal population variance assumption among the three types of wire rope. Why do you think this is? *Hint:* Think about how the F statistic is created.

(d) Sketch a new set of side-by-side boxplots (similar to the ones on the last page) that would produce a very large F statistic, maybe like $F \approx 17$ that we saw with the mortar data in class. Use the back of this page if necessary. *Note:* I like graphs that look nice.