5.7. The support is $R = \{(y_1, y_2) : y_1 > 0, y_2 > 0\}$, the entire first quadrant. See the picture below (left):

The joint pdf $f_{Y_1,Y_2}(y_1,y_2)$ is a three-dimensional function which takes the value $e^{-(y_1+y_2)}$ over this region (i.e., the entire first quadrant) and is otherwise equal to zero.

(a) We calculate $P(Y_1 < 1, Y_2 > 5)$ by integrating the joint pdf $f_{Y_1,Y_2}(y_1,y_2)$ over the set

$$B = \{(y_1, y_2) : y_1 < 1, y_2 > 5\}.$$

This set is shown above (right). The limits come from the picture. Therefore,

$$P(Y_1 < 1, Y_2 > 5) = \int_{y_1 = 0}^{1} \int_{y_2 = 5}^{\infty} e^{-(y_1+y_2)} dy_2 dy_1 = \int_{y_1 = 0}^{1} e^{-y_1} \left( \int_{y_2 = 5}^{\infty} e^{-y_2} dy_2 \right) dy_1$$

$$= \int_{y_1 = 0}^{1} e^{-y_1} \left[ \left( -e^{-y_2} \right)_{y_2 = 5}^{\infty} \right] dy_1$$

$$= \int_{y_1 = 0}^{1} e^{-y_1} (0 + e^{-5}) dy_1$$

$$= e^{-5} \int_{y_1 = 0}^{1} e^{-y_1} dy_1$$

$$= e^{-5} \left( -e^{-y_1} \right)_{y_1 = 0}^{1} = e^{-5} (1 - e^{-1}) \approx 0.004.$$

This is the (approximate) volume under $f_{Y_1,Y_2}(y_1,y_2)$ over the set $B$ above.

(b) We calculate $P(Y_1 + Y_2 < 3)$ by integrating the joint pdf $f_{Y_1,Y_2}(y_1,y_2)$ over the set

$$B = \{(y_1, y_2) : 0 < y_1 + y_2 < 3\}.$$

This set is shown on the next page (top). Note that the boundary of this set is

$$y_1 + y_2 = 3 \implies y_2 = 3 - y_1,$$

a linear function of $y_1$ with slope $-1$ and intercept $3$. 

---

PAGE 1
The limits come from the picture. Therefore,

\[ P(Y_1 + Y_2 < 3) = \int_{y_1=0}^{3} \int_{y_2=0}^{3-y_1} e^{-(y_1+y_2)} \, dy_2 \, dy_1 = \int_{y_1=0}^{3} e^{-y_1} \left( \int_{y_2=0}^{3-y_1} e^{-y_2} \, dy_2 \right) \, dy_1 \]

\[ = \int_{y_1=0}^{3} e^{-y_1} \left( -e^{-y_2} \bigg|_{y_2=0}^{3-y_1} \right) \, dy_1 \]

\[ = \int_{y_1=0}^{3} e^{-y_1} \left( 1 - e^{-3+y_1} \right) \, dy_1 \]

\[ = \int_{y_1=0}^{3} (e^{-y_1} - e^{-3}) \, dy_1 \]

\[ = \left( -e^{-y_1} - e^{-3}y_1 \right) \bigg|_{y_1=0}^{3} \]

\[ = -[(e^{-3}+3e^{-3}) - (1+0)] = 1 - 4e^{-3} \approx 0.801. \]

This is the (approximate) volume under \( f_{Y_1,Y_2}(y_1, y_2) \) over the set \( B \) above.

5.9. The support \( R = \{(y_1, y_2) : 0 \leq y_1 \leq y_2 \leq 1\} \) is a triangular region. See the picture on the next page (left). The joint pdf \( f_{Y_1,Y_2}(y_1, y_2) \) is a three-dimensional function which takes the value \( k(1 - y_2) \) over this region and is otherwise equal to zero.

(a) We know the joint pdf \( f_{Y_1,Y_2}(y_1, y_2) \) integrates to 1 over the support. The limits come from the picture. Therefore,

\[ 1 = \int_{y_2=0}^{1} \int_{y_1=0}^{y_2} k(1 - y_2) \, dy_1 \, dy_2 = k \int_{y_2=0}^{1} (1 - y_2) \left( y_1 \bigg|_{y_1=0}^{y_2} \right) \, dy_2 \]

\[ = k \int_{y_2=0}^{1} y_2(1 - y_2) \, dy_2 = k \frac{\Gamma(2)\Gamma(2)}{\Gamma(4)} = \frac{k}{6} \implies k = 6. \]

Note that the integrand in \( \int_{y_2=0}^{1} y_2(1 - y_2) \, dy_2 \) is the kernel of the beta\((2, 2)\) pdf and we are integrating over \((0, 1)\). Therefore, this integral can be done quickly.
(b) The joint pdf of $Y_1$ and $Y_2$ is
\[
f_{Y_1,Y_2}(y_1, y_2) = \begin{cases} 
6(1 - y_2), & 0 \leq y_1 \leq y_2 \leq 1 \\
0, & \text{otherwise}.
\end{cases}
\]

We calculate $P(Y_1 \leq 3/4, Y_2 \geq 1/2)$ by integrating the joint pdf $f_{Y_1,Y_2}(y_1, y_2)$ over the set
\[
B = \{(y_1, y_2) : y_1 \leq 3/4, \ y_2 \geq 1/2\}.
\]

This set is shown above (right). The limits come from the picture. We have to break this up into two integrals:
\[
\int_{y_2=3/4}^{1} \int_{y_1=0}^{3/4} 6(1 - y_2) \, dy_1 \, dy_2 \quad \text{over upper rectangular region}
\]
and
\[
\int_{y_2=1/2}^{3/4} \int_{y_1=0}^{y_2} 6(1 - y_2) \, dy_1 \, dy_2 \quad \text{over lower trapezoidal region}
\]

We do both integrals and then add them. The first integral is
\[
\int_{y_2=3/4}^{1} 6 \left(1 - y_2\right) \left(\frac{3}{4}\right) \, dy_2 = 6 \left(\frac{3}{4}\right) \left(y_2 - \frac{y_2^2}{2}\right)_{y_2=3/4}^{1} = 6 \left(\frac{3}{4}\right) \left(1 - \frac{1}{2} - \frac{3}{4} + \frac{9}{32}\right) \approx 0.141.
\]
The second integral is
\[
\int_{y_2=1/2}^{3/4} \int_{y_1=0}^{y_2} 6 \left(1 - y_2\right) \, dy_1 \, dy_2 = \int_{y_2=1/2}^{3/4} 6y_2 \left(1 - y_2\right) \, dy_1 \, dy_2 = 6 \left(\frac{y_2^2}{2} - \frac{y_2^3}{3}\right)_{y_2=1/2}^{3/4} = 6 \left(\frac{9}{32} - \frac{27}{192} - \frac{1}{8} + \frac{1}{24}\right) \approx 0.344.
\]

Therefore, $P(Y_1 \leq 3/4, Y_2 \geq 1/2) \approx 0.141 + 0.344 = 0.485$. This is the (approximate) volume under $f_{Y_1,Y_2}(y_1, y_2)$ over the set $B$ above.
5.10. The support
\[ R = \{(y_1, y_2) : 0 \leq y_1 \leq 2, 0 \leq y_2 \leq 1, y_1 \geq 2y_2\} \]
is the triangular region in the picture above (left). Note that the boundary of this support is
\[ y_1 = 2y_2 \implies y_2 = \frac{y_1}{2}, \]
a linear function of \( y_1 \) with slope \( 1/2 \) and intercept \( 0 \). The joint pdf \( f_{Y_1,Y_2}(y_1, y_2) \) is a three-dimensional function which takes the value \( k \) over this region and is otherwise equal to zero. In other words, the joint pdf \( f_{Y_1,Y_2}(y_1, y_2) \) is constant (with height \( k \)) over this triangle.

(a) We know the joint pdf \( f_{Y_1,Y_2}(y_1, y_2) \) integrates to 1 over the support. We could write
\[ 1 = \int_{y_1=0}^{2} \int_{y_2=0}^{y_1/2} k \, dy_2 dy_1 \]
and solve for \( k \). However, we could also find \( k \) from using elementary geometry. The area of the triangle (base/support) is
\[ \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 1 \times 1 = 1. \]
Therefore, the height of the function \( f_{Y_1,Y_2}(y_1, y_2) \) must be 1 in order for the volume under \( f_{Y_1,Y_2}(y_1, y_2) \) to equal 1. Therefore, \( k = 1 \). If you solve the integral equation above for \( k \), you will also get \( k = 1 \).

(b) We calculate \( P(Y_1 \geq 3Y_2) \) by integrating the joint pdf \( f_{Y_1,Y_2}(y_1, y_2) \) over the set
\[ B = \{(y_1, y_2) : y_1 \geq 3y_2\}. \]
This set is shown above (right). Note that the boundary of \( B \) is
\[ y_1 = 3y_2 \implies y_2 = \frac{y_1}{3}, \]
a linear function of $y_1$ with slope $1/3$ and intercept 0. The limits come right from this picture; i.e., we could calculate

$$P(Y_1 \geq 3Y_2) = \int_{y_1=0}^{2/3} \int_{y_2=0}^{y_1/3} 1 \, dy_2 \, dy_1.$$ 

However, again because the height of the joint pdf $f_{Y_1,Y_2}(y_1,y_2) = 1$ (constant), we can calculate this volume quickly using geometry again:

$$\text{volume} = \frac{1}{2} \times \text{base} \times \text{height} \times \text{height of pdf} = \frac{1}{2} \times 2 \times \frac{2}{3} \times 1 = \frac{2}{3}.$$ 

You would get the same answer if you did the double integral above.

5.16. The support is $R = \{(y_1, y_2) : 0 \leq y_1 \leq 1, 0 \leq y_2 \leq 1\}$, the unit square. See below:

![Diagram](image)

The joint pdf $f_{Y_1,Y_2}(y_1,y_2)$ is a three-dimensional function which takes the value $y_1 + y_2$ over this region and is otherwise equal to zero.

(a) We calculate $P(Y_1 < 1/2, Y_2 > 1/4)$ by integrating the joint pdf $f_{Y_1,Y_2}(y_1,y_2)$ over the set $B = \{(y_1, y_2) : y_1 < 1/2, y_2 > 1/4\}$.

This set is shown on the next page (left). The limits come from the picture. Therefore,

$$P(Y_1 < 1/2, Y_2 > 1/4) = \int_{y_2=1/4}^{1} \int_{y_1=0}^{1/2} (y_1 + y_2) \, dy_1 \, dy_2$$

$$= \int_{y_2=1/4}^{1} \left( \frac{y_2^2}{2} + y_1 y_2 \right) \bigg|_{y_1=0}^{1/2} \, dy_2$$

$$= \int_{y_2=1/4}^{1} \left( \frac{1}{8} + \frac{y_2}{2} \right) \, dy_2$$

$$= \left( \frac{y_2^2}{8} + \frac{y_2^2}{4} \right) \bigg|_{y_2=1/4}^{1} = \frac{1}{8} + \frac{1}{4} - \frac{1}{32} - \frac{1}{64} = \frac{21}{64} \approx 0.328.$$ 

This is the volume under $f_{Y_1,Y_2}(y_1,y_2)$ over the set $B$ above (picture on next page).
(b) We calculate $P(Y_1 + Y_2 \leq 1)$ by integrating the joint pdf $f_{Y_1,Y_2}(y_1, y_2)$ over the set 

$$B = \{(y_1, y_2) : y_1 + y_2 \leq 1\}.$$ 

This set is shown above (right). Note that the boundary of $B$ is 

$$y_1 + y_2 = 1 \implies y_2 = 1 - y_1,$$

a linear function of $y_1$ with slope $-1$ and intercept 1. Therefore,

$$P(Y_1 + Y_2 \leq 1) = \int_{y_1=0}^{1} \int_{y_2=0}^{1-y_1} (y_1 + y_2) \, dy_1 \, dy_2$$

$$= \int_{y_1=0}^{1} \left( y_1 y_2 + \frac{y_2^2}{2} \right) \bigg|_{y_2=0}^{1-y_1} \, dy_1$$

$$= \int_{y_1=0}^{1} \left[ y_1(1 - y_1) + \frac{(1 - y_1)^2}{2} \right] \, dy_1$$

$$= \int_{y_1=0}^{1} y_1(1 - y_1) \, dy_1 + \frac{1}{2} \int_{y_1=0}^{1} (1 - y_1)^2 \, dy_1$$

$$= \frac{\Gamma(2)\Gamma(2)}{\Gamma(4)} + \frac{1}{2} \frac{\Gamma(1)\Gamma(3)}{\Gamma(4)} = \frac{\frac{3}{2}}{6} + \frac{1}{2} \left( \frac{1}{3} \right) = \frac{1}{3}.$$

This is the volume under $f_{Y_1,Y_2}(y_1, y_2)$ over the set $B$ above. Note that the integrand in the first integral is a beta(2, 2) kernel. The integrand in the second integral is a beta(1, 3) kernel. Both integrals are over $(0, 1)$. Therefore, these integrals can be done quickly.

5.18. The support is $R = \{(y_1, y_2) : y_1 > 0, y_2 > 0\}$, the entire first quadrant. See the picture on the next page (left). We calculate $P(Y_1 > 1, Y_2 > 1)$ by integrating the joint pdf $f_{Y_1,Y_2}(y_1, y_2)$
over the set

\[ B = \{ (y_1, y_2) : y_1 > 1, \ y_2 > 1 \} \]

This set is shown above (right). The limits come from the picture. Therefore,

\[
P(Y_1 > 1, Y_2 > 1) = \int_{y_2=1}^{\infty} \int_{y_1=1}^{\infty} \frac{1}{8} y_1 e^{-(y_1+y_2)/2} \, dy_1 \, dy_2
\]

\[
= \frac{1}{8} \int_{y_2=1}^{\infty} e^{-y_2/2} \left( \int_{y_1=1}^{\infty} y_1 e^{-y_1/2} \, dy_1 \right) \, dy_2.
\]

Let’s do the inner integral by parts:

\[
u = y_1, \quad du = dy_1 \quad dv = e^{-y_1/2}, \quad v = -2e^{-y_1/2}.
\]

Therefore, the inner integral is

\[
\int_{y_1=1}^{\infty} y_1 e^{-y_1/2} \, dy_1 = -2y_1 e^{-y_1/2} \Bigg|_{y_1=1}^{\infty} - \int_{y_1=1}^{\infty} 2e^{-y_1/2} \, dy_1
\]

\[
= \left( 0 + 2e^{-1/2} \right) - 2 \left( 2e^{-1/2} \right) \Bigg|_{y_1=1}^{\infty}
\]

\[
= 2e^{-1/2} + 2 \left( 2e^{-1/2} - 0 \right) = 6e^{-1/2}.
\]

Finally,

\[
P(Y_1 > 1, Y_2 > 1) = \frac{6e^{-1/2}}{8} \int_{y_2=1}^{\infty} e^{-y_2/2} \, dy_2
\]

\[
= \frac{6e^{-1/2}}{8} \left( -2e^{-y_2/2} \right) \Bigg|_{y_2=1}^{\infty} = \frac{6e^{-1/2}}{8} \left( 2e^{-1/2} - 0 \right) = \frac{12e^{-1}}{8} \approx 0.552.
\]

This is the volume under \( f_{Y_1,Y_2}(y_1, y_2) \) over the set \( B \) above.
5.22. The table below shows the joint pmf of $Y_1$ and $Y_2$.

<table>
<thead>
<tr>
<th>$P_{Y_1,Y_2}(y_1,y_2)$</th>
<th>$y_1 = 0$</th>
<th>$y_1 = 1$</th>
<th>$P_{Y_2}(y_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_2 = 0$</td>
<td>0.38</td>
<td>0.17</td>
<td>0.55</td>
</tr>
<tr>
<td>$y_2 = 1$</td>
<td>0.14</td>
<td>0.02</td>
<td>0.16</td>
</tr>
<tr>
<td>$y_2 = 2$</td>
<td>0.24</td>
<td>0.05</td>
<td>0.29</td>
</tr>
<tr>
<td>$P_{Y_1}(y_1)$</td>
<td>0.76</td>
<td>0.24</td>
<td></td>
</tr>
</tbody>
</table>

(a) The marginal pmfs are in the margins; i.e., the marginal pmf of $Y_1$ is

\[
\begin{array}{c|cc}
   y_1 & 0 & 1 \\
\hline
   P_{Y_1}(y_1) & 0.76 & 0.24 \\
\end{array}
\]

Note that $Y_1 \sim \text{Bernoulli}(p = 0.24)$. The marginal pmf of $Y_2$ is

\[
\begin{array}{c|ccc}
   y_2 & 0 & 1 & 2 \\
\hline
   P_{Y_2}(y_2) & 0.55 & 0.16 & 0.29 \\
\end{array}
\]

(b) The conditional pmf $P_{Y_2|Y_1}(y_2|y_1 = 0)$ describes the distribution of $Y_2$ when $Y_1 = 0$. This is a univariate pmf with three possible values of $Y_2$, namely, 0, 1, and 2. These conditional probabilities are calculated below:

\[
\begin{align*}
P_{Y_2|Y_1}(y_2 = 0|y_1 = 0) &= \frac{P_{Y_1,Y_2}(0,0)}{P_{Y_1}(0)} = \frac{0.38}{0.76} = 0.50 \\
P_{Y_2|Y_1}(y_2 = 1|y_1 = 0) &= \frac{P_{Y_1,Y_2}(0,1)}{P_{Y_1}(0)} = \frac{0.14}{0.76} \approx 0.184 \\
P_{Y_2|Y_1}(y_2 = 2|y_1 = 0) &= \frac{P_{Y_1,Y_2}(0,2)}{P_{Y_1}(0)} = \frac{0.24}{0.76} \approx 0.316.
\end{align*}
\]

We can display the conditional pmf of $Y_2$ given $Y_1 = 0$ in the following table:

\[
\begin{array}{c|ccc}
   y_2 & 0 & 1 & 2 \\
\hline
   P_{Y_2|Y_1}(y_2|y_1 = 0) & 0.50 & 0.184 & 0.316 \\
\end{array}
\]

(c) We want $P(Y_1 = 0|Y_2 = 2)$. We can calculate this as follows:

\[
P(Y_1 = 0|Y_2 = 2) = \frac{P(Y_1 = 0, Y_2 = 2)}{P(Y_2 = 2)} = \frac{P_{Y_1,Y_2}(0,2)}{P_{Y_2}(2)} = \frac{0.24}{0.29} \approx 0.828.
\]

5.26. The support is $R = \{(y_1,y_2) : 0 \leq y_1 \leq 1, 0 \leq y_2 \leq 1\}$, the unit square; see next page (left). The joint pdf $f_{Y_1,Y_2}(y_1,y_2)$ is a three-dimensional function which takes the value $4y_1y_2$ over this region and is otherwise equal to zero.

(a) The marginal pdf of $Y_1$ is nonzero when $0 \leq y_1 \leq 1$. For these values, the pdf is

\[
f_{Y_1}(y_1) = \int_{\mathbb{R}} f_{Y_1,Y_2}(y_1,y_2) \, dy_2 = \int_{y_2=0}^{1} 4y_1y_2 \, dy_2 = 4y_1 \left( \frac{y_2^2}{2} \right)_{y_2=0}^{1} = 2y_1.
\]
Therefore, the marginal pdf of $Y_1$ is given by

$$f_{Y_1}(y_1) = \begin{cases} 2y_1, & 0 \leq y_1 \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

You should recognize this as a beta pdf with parameters $\alpha = 2$ and $\beta = 1$. That is, marginally, $Y_1 \sim \text{beta}(2, 1)$. The exact same argument shows that $Y_2 \sim \text{beta}(2, 1)$; i.e., the pdf of $Y_2$ is

$$f_{Y_2}(y_2) = \begin{cases} 2y_2, & 0 \leq y_2 \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

**Note:** You should see that $Y_1$ and $Y_2$ are independent because

$$f_{Y_1,Y_2}(y_1, y_2) = f_{Y_1}(y_1)f_{Y_2}(y_2).$$

However, the notion of independence has not been introduced so far at this point in the textbook, so we will carry out all future calculations as is.

(b) This is a conditional probability:

$$P(Y_1 \leq 1/2 | Y_2 \geq 3/4) = \frac{P(Y_1 \leq 1/2, Y_2 \geq 3/4)}{P(Y_2 \geq 3/4)}.$$ 

We calculate the numerator probability $P(Y_1 \leq 1/2, Y_2 \geq 3/4)$ using the joint pdf of $Y_1$ and $Y_2$ and integrating it over the set $B = \{(y_1, y_2) : y_1 \leq 1/2, y_2 \geq 3/4\}$; see above (right):

$$P(Y_1 \leq 1/2, Y_2 \geq 3/4) = \int_{y_1=0}^{1/2} \int_{y_2=3/4}^{1} 4y_1y_2 \, dy_2 \, dy_1 = 4 \int_{y_1=0}^{1/2} y_1 \left( \frac{y_2^2}{2} \right)_{y_2=3/4}^{1} \, dy_1 = 4 \left( \frac{1}{2} - \frac{9}{32} \right) \left( \frac{y_1^2}{2} \right)_{y_1=0}^{1/2} = 4 \left( \frac{7}{32} \right) \frac{1}{8} = \frac{7}{64}.$$
We can calculate the denominator probability \( P(Y_2 \geq 3/4) \) using either the marginal pdf of \( Y_2 \) or the joint pdf of \( Y_1 \) and \( Y_2 \). Let’s use the marginal because we already have it:

\[
P(Y_2 \geq 3/4) = \int_{y_2=3/4}^{1} 2y_2 \, dy_2 = y_2^2 \bigg|_{y_2=3/4}^{1} = 1 - \frac{9}{16} = \frac{7}{16}.
\]

Therefore,

\[
P(Y_1 \leq 1/2|Y_2 \geq 3/4) = \frac{7/64}{7/16} = \frac{1}{4}.
\]

**Note:** Because \( Y_1 \) and \( Y_2 \) are independent, note that

\[
P(Y_1 \leq 1/2|Y_2 \geq 3/4) = P(Y_1 \leq 1/2) = \int_{y_1=0}^{1/2} 2y_1 \, dy_1 = y_1^2 \bigg|_{y_1=0}^{1/2} = \frac{1}{4}.
\]

Independence makes are lives easier!

(c) Whatever the value of \( Y_2 = y_2 \) is, the possible values of \( Y_1 \) are \( 0 \leq y_1 \leq 1 \); see the bivariate support on the last page. For these values, the conditional pdf of \( Y_1 \) is

\[
f_{Y_1|Y_2}(y_1|y_2) = \frac{f_{Y_1,Y_2}(y_1,y_2)}{f_{Y_2}(y_2)} = \frac{4y_1y_2}{2y_2} = 2y_1.
\]

Summarizing,

\[
f_{Y_1|Y_2}(y_1|y_2) = \begin{cases} 
2y_1, & 0 \leq y_1 \leq 1 \\
0, & \text{otherwise}.
\end{cases}
\]

**Note:** The conditional pdf \( f_{Y_1|Y_2}(y_1|y_2) \) is the same as the marginal pdf

\[
f_{Y_1}(y_1) = \begin{cases} 
2y_1, & 0 \leq y_1 \leq 1 \\
0, & \text{otherwise}.
\end{cases}
\]

This makes sense because \( Y_1 \) and \( Y_2 \) are independent.

(d) This is completely analogous to part (c). The same argument shows

\[
f_{Y_2|Y_1}(y_2|y_1) = \begin{cases} 
2y_2, & 0 \leq y_2 \leq 1 \\
0, & \text{otherwise},
\end{cases}
\]

which is the same as the marginal pdf of \( Y_2 \).

(e) We calculate \( P(Y_1 \leq 3/4|Y_2 = 1/2) \) using the conditional pdf of \( Y_1 \) when \( Y_2 = 1/2 \); see the formula for \( f_{Y_1|Y_2}(y_1|y_2) \) above. As we have already seen, the formula for \( f_{Y_1|Y_2}(y_1|y_2) \) does not depend on \( y_2 \) (because of independence). We have

\[
P(Y_1 \leq 3/4|Y_2 = 1/2) = \int_{y_1=0}^{3/4} f_{Y_1|Y_2}(y_1|y_2 = 3/4) \, dy_1 = \int_{y_1=0}^{3/4} 2y_1 \, dy_1 = y_1^2 \bigg|_{y_1=0}^{3/4} = \frac{9}{16}.
\]

Note that this is the same answer you would have gotten had you just calculated \( P(Y_1 \leq 3/4) \) using the marginal pdf of \( Y_1 \). This is true because \( Y_1 \) and \( Y_2 \) are independent.
5.32.