Below are some distributional relationships we have discovered. Also see Leemis and McQueston (2008). The symbol $\sum Y_i$ is understood to mean $\sum_{i=1}^n Y_i$.

1. If $Y \sim \mathcal{N}(\mu, \sigma^2)$, then
$$Z = \frac{Y - \mu}{\sigma} \sim \mathcal{N}(0,1)$$

2. $Y \sim \mathcal{N}(0,1) \Longrightarrow Y^2 \sim \chi^2(1)$

3. $Y \sim \mathcal{N}(\mu, \sigma^2) \Longrightarrow aY + b \sim \mathcal{N}(a\mu + b, a^2\sigma^2)$

4. $Y \sim \mathcal{U}(0,1) \Longrightarrow -\ln Y \sim \text{exponential}(1)$
<u>Generalization</u>: $Y \sim \mathcal{U}(0,1) \Longrightarrow -\beta \ln Y \sim \text{exponential}(\beta)$
<u>Related</u>: $Y \sim \text{beta}(\alpha, 1) \Longrightarrow -\ln Y \sim \text{exponential}(1/\alpha)$
<u>Related</u>: $Y \sim \text{beta}(1, \beta) \Longrightarrow -\ln(1-Y) \sim \text{exponential}(1/\beta)$

5. $Y \sim \text{exponential}(\alpha) \Longrightarrow Y^{1/m} \sim \text{Weibull}(m, \alpha)$
<u>Related</u>: $Y \sim \text{Weibull}(m, \alpha) \Longrightarrow Y^m \sim \text{exponential}(\alpha)$

6. $Y \sim \mathcal{N}(\mu, \sigma^2) \Longrightarrow e^Y \sim \text{lognormal}(\mu, \sigma^2)$ or equivalently if $U \sim \text{lognormal}(\mu, \sigma^2) \Longrightarrow \ln U \sim \mathcal{N}(\mu, \sigma^2)$

7. $Y \sim \text{beta}(\alpha, \beta) \Longrightarrow 1 - Y \sim \text{beta}(\beta, \alpha)$

8. $Y \sim \mathcal{U}(-\pi/2, \pi/2) \Longrightarrow \tan Y \sim \text{Cauchy}$

9. $Y \sim \text{gamma}(\alpha, \beta) \Longrightarrow cY \sim \text{gamma}(\alpha, \beta c)$, where $c > 0$
<u>Special case</u>: $2Y/\beta \sim \chi^2(2\alpha)$

10. $Y_1, Y_2, ..., Y_n \sim \text{iid Bernoulli}(p) \Longrightarrow \sum Y_i \sim b(n, p)$

11. $Y_i \sim \text{gamma}(\alpha_i, \beta), i = 1, 2, ..., n$ (mutually independent)
$$\Longrightarrow \sum Y_i \sim \text{gamma}\left(\sum \alpha_i, \beta\right)$$
<u>Special case</u>: $\alpha_i = 1$, for $i = 1, 2, ..., n$. Then $Y_1, Y_2, ..., Y_n \sim \text{iid exponential}(\beta) \Longrightarrow \sum Y_i \sim \text{gamma}(n, \beta)$
<u>Special case</u>: $\alpha_i = \nu_i/2$, $\beta = 2$. Then $Y_i \sim \chi^2(\nu_i), i = 1, 2, ..., n$ (mutually independent) $\Longrightarrow \sum Y_i \sim \chi^2(\sum \nu_i)$
<u>Combination</u>: If $Y_1, Y_2, ..., Y_n \sim \text{iid exponential}(\beta)$, then
$$\frac{2\sum Y_i}{\beta} \sim \chi^2(2n)$$

12. $Y_i \sim \text{Poisson}(\lambda_i), i = 1, 2, ..., n$ (mutually independent)
$$\Longrightarrow \sum Y_i \sim \text{Poisson}\left(\sum \lambda_i\right)$$

13. $Y_i \sim \mathcal{N}(\mu_i, \sigma_i^2), i = 1, 2, ..., n$ (mutually independent)

$$\Longrightarrow \sum a_i Y_i \sim \mathcal{N}\left(\sum a_i \mu_i, \sum a_i^2 \sigma_i^2\right)$$

Special case: $\mu_i = \mu$ and $\sigma_i^2 = \sigma^2$, for $i = 1, 2, ..., n$. Then $Y_1, Y_2, ..., Y_n \sim$ iid $\mathcal{N}(\mu, \sigma^2)$

$$\Longrightarrow \sum a_i Y_i \sim \mathcal{N}\left(\mu \sum a_i, \sigma^2 \sum a_i^2\right)$$

Special case of iid result: If $Y_1, Y_2, ..., Y_n \sim$ iid $\mathcal{N}(\mu, \sigma^2)$, then $\overline{Y} \sim \mathcal{N}(\mu, \sigma^2/n)$
Special case of iid result: If $Y_1, Y_2, ..., Y_n \sim$ iid $\mathcal{N}(\mu, \sigma^2)$, then $\sum Y_i \sim \mathcal{N}(n\mu, n\sigma^2)$

14. If $Y_i \sim \mathcal{N}(\mu_i, \sigma_i^2), i = 1, 2, ..., n$ (mutually independent), then

$$Z_i = \frac{Y_i - \mu_i}{\sigma_i} \sim \mathcal{N}(0, 1),$$

for $i = 1, 2, ..., n$. Therefore, $U = \sum Z_i^2 \sim \chi^2(n)$ because $Z_1^2, Z_2^2, ..., Z_n^2$ are iid $\chi^2(1)$

15. $Y_1, Y_2, ..., Y_n \sim$ iid geometric$(p) \Longrightarrow U = \sum Y_i \sim$ nib(n, p)

16. $Y_1, Y_2 \sim$ iid $\mathcal{N}(0, 1) \Longrightarrow U = Y_1/Y_2 \sim$ Cauchy

17. $Y_1, Y_2, ..., Y_n \sim$ iid exponential$(\beta) \Longrightarrow Y_{(1)} \sim$ exponential(β/n)

18. $Y_1, Y_2, ..., Y_n \sim$ iid Weibull$(m, \alpha) \Longrightarrow Y_{(1)} \sim$ Weibull$(m, \alpha/n)$

19. If $Z \sim \mathcal{N}(0, 1)$, $W \sim \chi^2(\nu)$, and $Z \perp\!\!\!\perp W$, then

$$T = \frac{Z}{\sqrt{W/\nu}} \sim t(\nu)$$

20. $Y_1, Y_2, ..., Y_n \sim$ iid $\mathcal{N}(\mu, \sigma^2)$

$$\Longrightarrow \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

21. If $W_1 \sim \chi^2(\nu_1)$, $W_2 \sim \chi^2(\nu_2)$, and $W_1 \perp\!\!\!\perp W_2$, then

$$F = \frac{W_1/\nu_1}{W_2/\nu_2} \sim F(\nu_1, \nu_2)$$

22. If $F \sim F(\nu_1, \nu_2)$, then $1/F \sim F(\nu_2, \nu_1)$

23. If $T \sim t(\nu)$, then $T^2 \sim F(1, \nu)$

24. If $W \sim F(\nu_1, \nu_2)$, then

$$\frac{(\nu_1/\nu_2)W}{1 + (\nu_1/\nu_2)W} \sim \text{beta}(\nu_1/2, \nu_2/2)$$