From WMS, do 10.2, 10.3, and 10.5. Also, complete the following extra problems (EP).

EP1. Suppose that $Y_1, Y_2, ..., Y_n$ is an iid $\mathcal{N}(\mu, \sigma_0^2)$ sample, where $\sigma_0^2 = 9$. We would like to test

$$H_0 : \mu = 10$$
versus
$$H_a : \mu > 10.$$

We will use a rejection region of the form $\text{RR} = \{\overline{y} : \overline{y} > k\}$. Find the values of n and k that provide a Type I Error probability of $\alpha = 0.02$ and a Type II Error probability of $\beta = 0.10$ when $\mu = 12$.

EP2. Suppose that $Y_1, Y_2, ..., Y_n$ is an iid sample of size n from $f_Y(y; \theta)$, where

$$f_Y(y;\theta) = \begin{cases} \theta y^{\theta-1}, & 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

We are interested in testing $H_0 : \theta = 1$ versus $H_a : \theta < 1$. To perform the test, suppose we use the test statistic

$$T = \prod_{i=1}^{n} Y_i$$

and the rejection region $\text{RR} = \{t : t < k\}$, where k is a constant.
(a) Show that T is a sufficient statistic for θ.
(b) Find an expression for k that makes RR have Type I Error probability equal to α.
Hint: Show that $U = -2\ln T \sim \chi^2(2n)$ when H_0 is true.

EP3. Suppose that $Y_1, Y_2, ..., Y_n$ is an iid $\mathcal{N}(0, \sigma^2)$ sample, where $\sigma^2 > 0$ is unknown. We are interested in testing

$$H_0 : \sigma^2 = \sigma_0^2$$
versus
$$H_a : \sigma^2 > \sigma_0^2,$$

where σ_0^2 is known. To perform the test, suppose we use the test statistic

$$T = \sum_{i=1}^{n} Y_i^2$$

and the rejection region $\text{RR} = \{t : t > k\}$, where k is a constant.
(a) Show that T is a sufficient statistic for σ^2.
(b) Determine the value of k that makes RR have Type I Error probability equal to α.
(c) Using your answer from part (b), find an expression for $\beta(\sigma_a^2)$, for $\sigma_a^2 \geq \sigma_0^2$. Show that $\beta(\sigma_a^2)$ is a decreasing function of σ_a^2.