From WMS, do 11.4, 11.10, 11.11, 11.15, and 11.20. Also, complete the following extra problem (EP).

EP1. Consider the simple linear regression model

\[ Y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \]

for \( i = 1, 2, \ldots, n \), where \( \epsilon_i \sim \text{iid } \mathcal{N}(0, \sigma^2) \).

(a) Show that the sum of the residuals \( e_i = Y_i - \hat{Y}_i \) from a least-squares fit equals zero; that is, show algebraically that

\[ \sum_{i=1}^{n} (Y_i - \hat{Y}_i) = 0. \]

(b) Prove that

\[ e_i \sim \mathcal{N}[0, \sigma^2(1 - m_{ii})], \]

where

\[ m_{ii} = \frac{1}{n} + \frac{(x_i - \bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}. \]

Note that this implies the residuals (unlike the errors) do not have constant variance. In an applied course, \( m_{ii} \) might be called the leverage associated with the \( i \)th observation. Leverages can be helpful in classifying observations as outliers or not.