STAT509 Homework 6 Solution

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Problem 1

(a) Note that the sample mean $\bar{y} = 708.91$ and the sample standard deviation $s = 52.44$, then a 95% confidence interval for the population mean lifetime μ is

$$[\bar{y} \pm t_{n-1,\alpha/2}\frac{s}{\sqrt{n}}] = [694.0067, 723.8133]$$

where $n = 50, \alpha = 1 - 0.95 = 0.05$. We are 95 percent confident that the population mean lifetime is between 694 hours and 724 hours. Because the advertised mean lifetime 750 hours is not contained in the 95% confidence interval, we have sufficient evidence to conclude that the population mean lifetime is less than 750 hours. Therefore, we do not suggest customers carry on this purchasement.

(b) The normal qq plot indicates that the data mildly departures from the normality assumption. Nevertheless, the 95% confidence interval is based on t distribution which is robust to the normal assumption, therefore, the above proposed suggestion is still convincing.

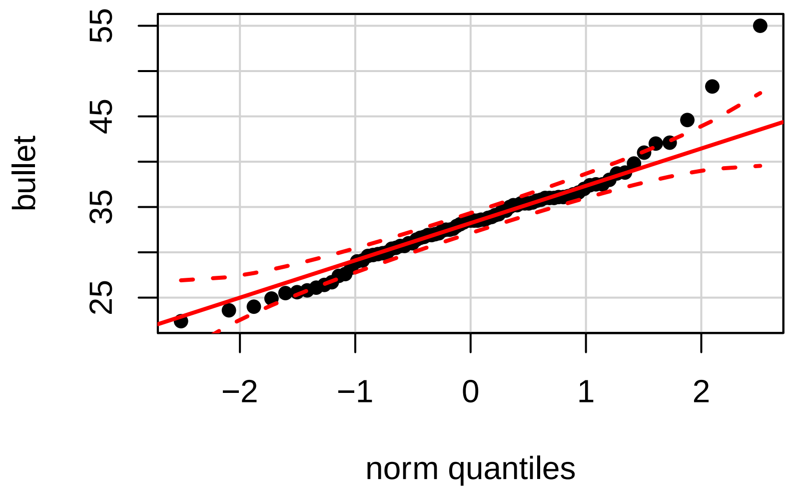
(c) Note that $n = 50, s^2 = 52.44^2 \approx 2749.9536$, the 95% confidence interval for σ is

$$[\sqrt{\frac{(n-1)s^2}{\chi^2_{n-1,\alpha/2}}}, \sqrt{\frac{(n-1)s^2}{\chi^2_{n-1,1-\alpha/2}}}] = [43.8049, 65.3472]$$

We are 95% confidence that the population standard deviation σ is between 44 and 65 hours. Because the confidence interval for σ critically depends the normal assumption, we are somewhat concerned because the normal qq plot indicating that the data mildly departures from the normal assumption.

Problem 2

```r
bullet=c(22.4, 23.6, 24.0, 24.9, 25.5, 25.6, 25.8, 26.1, 26.4, 26.7,
         27.4, 27.6, 28.3, 29.0, 29.1, 29.6, 29.7, 29.8, 29.9, 30.0,
         30.4, 30.5, 30.7, 30.7, 31.0, 31.0, 31.4, 31.6, 31.7, 31.9,
         31.9, 32.0, 32.1, 32.4, 32.5, 32.5, 32.6, 32.9, 33.1, 33.3,
         33.5, 33.5, 33.5, 33.5, 33.6, 33.6, 33.8, 33.9, 34.1, 34.2,
         34.6, 34.6, 35.0, 35.2, 35.2, 35.4, 35.4, 35.4, 35.5, 35.7,
         35.8, 36.0, 36.0, 36.0, 36.1, 36.1, 36.2, 36.4, 36.6, 37.0,
         37.4, 37.5, 37.5, 38.0, 38.7, 38.8, 39.8, 41.0, 42.0, 42.1,
         44.6, 48.3, 55.0)
library(car)
qqPlot(bullet,distribution = "norm",pch=16)
```



(a) Denote that Y = depth of a bullet penetrates ceramic body armor. Note that the sample mean $\bar{y}$ = 33.37 and the sample standard deviation $s = 5.27$. Then, the 95% confidence interval for the population mean depth of a bullet penetrates ceramic body armor μ is

$$[\bar{y} \pm t_{n-1,\alpha/2} \frac{s}{\sqrt{n}}] = [32.2193, 34.5207]$$

We are 95% confident that the population mean depth of a bullet penetrating ceramic body armor is between 32.22 and 34.52 mm. The normal qqplot indicates that the data mildly departures from the nomal assumption, but it does not bother us somewhat because t distribution is robust to the normal assumption.

(b) The 95% confident interval is an inference for the unknown population mean μ rather than for the observed data.

(c) Note that $n = 83, s^2 = 5.27^2$, the 95% confidence interval for σ is

$$[\sqrt{\frac{(n-1)s^2}{\chi^2_{n-1,\alpha/2}}}, \sqrt{\frac{(n-1)s^2}{\chi^2_{n-1,1-\alpha/2}}}] = [4.5722, 6.2211]$$

We are 95% confidence that the population standard deviation σ is between 4.57 and 6.22 mm. Because the confidence interval for σ critically depends the normal assumption, we are somewhat concerned because the normal qq plot indicating that the data mildly departures from the normal assumption.

Problem 3

(a) Denote that $p =$ population proportion of type A blood. Note that the sample proportion $\hat{p} = \frac{42}{80} = 0.525$ and the standard deviation of sample proportion $\hat{p}$ is $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, therefore a 95% confidence interval for p is

$$[\hat{p} \pm Z_{0.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}] = [0.4156, 0.6344]$$

We are 95% confident that the population proportion of type A blood is between 41.56% and 63.44%. Since the hypothesized population proportion $p = 0.4$ is below the 95% confidence interval, we have sufficient evidence to conclude that the percentage of type A donations differs from 40%. Because in the sample the number of type A blood is 42 and the number of non-type A blood is 38, which are both greater 5, the confidence interval above is valid.

(b) Note that the required margin of error $B = \frac{0.05}{2} = 0.025$ and $\alpha = 1 - 0.99 = 0.01$. And a best guess of the population proportion of type A Blood for the African-American population is $p_0 = 0.4$, then the optimal sample size for attaining the requirements is gained as follows.

$$n = (\frac{Z_{\alpha/2}}{B})^2 p_0(1-p_0) = 2556$$

Problem 4

(a) Note that $t = \frac{Y^* - \bar{Y}}{S\sqrt{1+1/n}} \sim t_{n-1}$, the $100(1-\alpha)$ confidence interval for the prediction of Y^* is

$$\begin{aligned} 1-\alpha &= P(-t_{n-1,\alpha/2} < \frac{Y^* - \bar{Y}}{S\sqrt{1+1/n}} < t_{n-1,\alpha/2}) \\ &= P(-t_{n-1,\alpha/2}S\sqrt{1+1/n} < Y^* - \bar{Y} < t_{n-1,\alpha/2}S\sqrt{1+1/n}) \\ &= P(\bar{Y} - t_{n-1,\alpha/2}S\sqrt{1+1/n} < Y^* < \bar{Y} + t_{n-1,\alpha/2}S\sqrt{1+1/n}) \end{aligned}$$

(b) For problem 1, the 95% prediction interval for Y^* is

$$[\bar{Y} \pm t_{n-1,\alpha/2}S\sqrt{1+1/n} = [601.4875, 816.5125]]$$

We are 95% confident that the lifetime for a new light bulb is between 601 and 817 hours.

(c) For the same confidence level, apparently the prediction interval for a new observation is wider than the interval for the population mean, because there is more uncertainty resulting from the new observation when it comes to derive the prediction interval.