STAT718A SHAPE AND IMAGE ANALYSIS Spring Session 2009

Homework 1

Date given out: 1/26/09. Please submit your solutions to the problems to me by 5.15pm February 4, 2009. This homework contributes 10% of the course grade.

1. Which of the triangles in Figure 1 have the same size and the same shape? (assume by 'shape' we mean labelled shape).

   Which triangles have the same shape but different sizes?

   Which triangles have different shapes but the same 'reflection shape'? i.e. a translation, rotation, scale and a reflection will match them exactly.

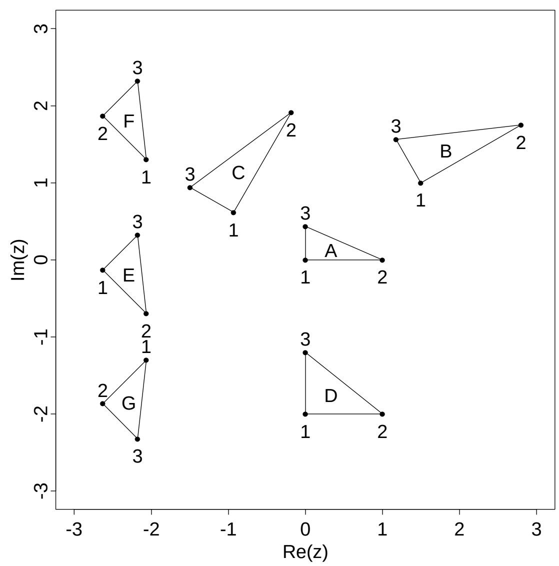

Figure 1: Some triangles

2. Rank the triangles in Figure 1 in order of increasing centroid size. Repeat the rankings using baseline size (with baseline 1, 2) and square root of area. [Note approximate calculations are fine.]

3. Informally rank the triangles in Figure 1 in order of 'closeness' to the anti-clockwise labelled equilateral triangle.

4. Calculate (approximately) the Bookstein shape coordinates for each triangle. Also, give the Kendall shape coordinates for each triangle.

/P.T.O.

5. In a forensic study photographs of the front view of the faces of alleged criminals are taken. Some landmarks are to be located on the face. For each of the following landmarks decide whether it is a Bookstein's Type I, II or II landmark. Also, state whether they are anatomical, mathematical or pseudo-landmarks.

   (a) Pupil of the eye
   (b) Tip of the nose
   (c) Corners of the mouth
   (d) Point on the cheek halfway between the pupil of the eye and corner of the mouth.
   (e) Centre of the forehead.
   (f) Lowest point on the chin.

   Which landmarks are easy to locate and which are difficult?

6. Prove the result that

$$
\begin{aligned}
\sum_{j=1}^{m}\sum_{i=1}^{k}\sum_{l=1}^{k}(X_{ij}-X_{lj})^2 &= 2k\sum_{i=1}^{k}\sum_{j=1}^{m}(X_{ij}-\bar{X}_j)^2 \\
&= 2kS(X)^2.
\end{aligned}
$$

7. Consider the Helmert sub-matrix H. Write down H for $k=3,4,5$ and verify that $H^{\mathrm{T}}H = C_k$ and $HH^{\mathrm{T}} = I_{k-1}$, where C_k is the $k \times k$ centring matrix.

   Prove the general results for $k \geq 2$ that $H^{\mathrm{T}}H = C_k$ and $HH^{\mathrm{T}} = I_{k-1}$