

Centerpoint Design Exercise

Background

Centerpoint designs add replications at the origin of a factorial design with *quantitative* factors; e.g., for a 2^2 design, centerpoints would be added at the design point (0,0). The additional observations at the centerpoint allow us to accomplish two goals:

1. Obtain an estimate of pure error (s_c^2 , the variance of the replications at the centerpoint)
2. Use the estimate of pure error to test for curvature

Of course, interaction is a type of curvature and we already know how to test for that. There are additional types of curvature, though, that we can test with the centerpoint model. We are particularly interested in testing for convex or concave shapes that would indicate the centerpoint is at a local maximum or minimum. In such cases, the mean of the centerpoint observations (\bar{y}_c) should be different from the mean of the factorial observations (\bar{y}_f). A simple t-test can be constructed to test for the difference; when analyzing the design, Minitab constructs the corresponding F-test for curvature. The t-test (with $n_c - 1$ degrees of freedom) has the form:

$$T = \frac{\bar{y}_c - \bar{y}_f}{s_c \sqrt{\frac{1}{n_c} + \frac{1}{n_f}}}$$

where n_f is the number of factorial points (this would be 2^k for a full factorial design).

Minitab macros

We will be using a Minitab macro to analyze a two-factor centerpoint design. The Minitab macro is available on the home page—you can either save it as a text file or cut and paste it into a text file—label it “rse.mac”. If you call the macro with a given set of factor levels (x-coordinates), the macro will evaluate an “unknown” function for you at those x-coordinates.

As an example, suppose you have the macro stored on subdirectory `mtb` on a diskette. If the design is stored in columns `c1` and `c2` and you want to store the response in `c3`, you would enter the following command in the Session window:

```
%a:\mtb\rse c1 c2 c3
```

You will use the macro as a tool in the following experiment. Please save output and data so we can discuss results in class.

Experiment

The bond strength between an integrated circuit and a substrate is a function of holding pressure and holding time. In a discussion with the process engineer, a reasonable start value for holding pressure is thought to be 40 to 44 psi while a reasonable start value for holding time is 20 to 25 minutes. Use this information to design an initial centerpoint

experiment in the two factors. Use the fractional factorial pull-down menus in Minitab to generate the appropriate design matrix. Select 5 centerpoint replications. Under **Factors**, change the **Low** and **High** factor levels as well as the factor **Names** accordingly. Call the rse macro to obtain your data. Analyze your data. Is there evidence of curvature? Compute the t-statistic for curvature by hand; is it the square root of the F-statistic produced by Minitab?

Follow-up experiment

The centerpoint design is just the first step in a sequential experimental process to find a maximum or minimum of a response. After the initial experiment fails to detect a maximum or minimum, we search in a promising direction (steepest ascent or steepest descent) until the response declines (increases). Once this occurs, we run another centerpoint design to confirm curvature and then run an augmented model to estimate the surface at the maximum (minimum).

Suppose a search from the initial experiment suggests that a promising set of experimental levels for the two factors would be (63,50). Select a centerpoint design with factor levels 53 and 73 psi for holding pressure and 40 to 60 for holding time; use 5 centerpoints again. Generate the response using the Minitab macro and analyze the data. Is there evidence of a local maximum? Confirm your conclusion with a contour plot of the response against the two factors.