Type III and Type IV Hypotheses Example

An education major wanted to test the efficacy of teaching methods for the division of fractions. Two new methods along with the standard method were studied. Five teachers were trained in all methods and taught a total of twelve classes. Differences in pre- and post-test scores are recorded below:

	Teacher				
Method	1	2	3	4	5
A	10, 7	6	11		6
B	4		5	7, 8	3
C		13	16		

We want to understand Type III and Type IV SS in SAS using this example. We can obtain the contrasts that SAS tests when testing Type III and Type IV SS using the following commands.

```
proc glm;
classes teacher method;
model diff=method|teacher/e e3 e4;
```

We obtain the following SAS output for the Type III SS contrasts for METHOD:

```
Type III Estimable Functions for:  METHOD

Effect                 Coefficients

INTERCEPT              0

METHOD            a    L2
                  b    L3
                  c    -L2-L3

TEACHER           1    0
                  2    0
                  3    0
                  4    0
                  5    0


TEACHER*METHOD 1 a    0.0909*L2-0.2273*L3
               1 b    -0.0909*L2+0.2273*L3
               2 a    0.4545*L2+0.3636*L3
               2 c    -0.4545*L2-0.3636*L3
               3 a    0.3636*L2+0.0909*L3
               3 b    0.1818*L2+0.5455*L3
               3 c    -0.5455*L2-0.6364*L3
```

```
4 b   0
5 a   0.0909*L2-0.2273*L3
5 b   -0.0909*L2+.2273*L3
```

The coefficients are parameter coefficients for the unconstrained model but they are used somewhat differently here than in the general form for estimable functions. The output above indicates that we are simultaneously testing whether the following two unusual contrasts are 0:

$$L_2 = \alpha_1 - \alpha_3 + .0909\gamma_{11} - .0909\gamma_{21} + .4545\gamma_{12} - .4545\gamma_{32} + .3636\gamma_{13} + .1818\gamma_{23}$$
$$-.5455\gamma_{33} + .0909\gamma_{15} - .0909\gamma_{25}$$

$$L_3 = \alpha_2 - \alpha_3 - .2273\gamma_{11} + .2273\gamma_{21} + .3636\gamma_{12} - .3636\gamma_{32} + .0909\gamma_{13} + .5455\gamma_{23}$$
$$-.6364\gamma_{33} - .2273\gamma_{15} + .2273\gamma_{25}$$

These can be reorganized into the following contrasts of cell means:

$$L_2 = \frac{1}{11}\mu_{11} + \frac{5}{11}\mu_{12} + \frac{4}{11}\mu_{13} + \frac{1}{11}\mu_{15}$$
$$-\frac{1}{11}\mu_{21} + \frac{2}{11}\mu_{23} - \frac{1}{11}\mu_{25} - \frac{5}{11}\mu_{32} - \frac{6}{11}\mu_{33}$$

$$L_3 = -\frac{5}{22}\mu_{11} + \frac{4}{11}\mu_{12} + \frac{1}{11}\mu_{13} - \frac{5}{22}\mu_{15}$$
$$+\frac{5}{22}\mu_{21} + \frac{6}{11}\mu_{23} + \frac{5}{22}\mu_{25} - \frac{4}{11}\mu_{32} - \frac{7}{11}\mu_{33}$$

Further reorganization sheds some light on these seemingly arbitrary contrasts:

$$L_2 = \frac{1}{11}(\mu_{11} - \mu_{21}) + \frac{5}{11}(\mu_{12} - \mu_{32})$$
$$+\left(\left(\frac{4}{11}\mu_{13} + \frac{2}{11}\mu_{23}\right) - \frac{6}{11}\mu_{33}\right) + \frac{1}{11}(\mu_{15} - \mu_{25})$$

$$L_3 = \frac{5}{22}(\mu_{21} - \mu_{11}) + \frac{4}{11}(\mu_{12} - \mu_{32})$$
$$+\left(\left(\frac{1}{11}\mu_{13} + \frac{6}{11}\mu_{23}\right) - \frac{7}{11}\mu_{33}\right) + \frac{5}{22}(\mu_{25} - \mu_{15})$$

With no missing cells, Type III SS should actually test the following hypotheses (regardless of whether the design is balanced):

$$PMM(\alpha_1) - PMM(\alpha_3) = 0,\ PMM(\alpha_2) - PMM(\alpha_3) = 0$$

You could add data to the missing cells to verify the previous statement. A quick glance at the Type IV contrasts shows that all problematic columns are eliminated when constructing the contrasts:

```
Type IV Estimable Functions for:  METHOD

Effect                  Coefficients

INTERCEPT               0

METHOD              a   L2
                    b   L3
                    c   -L2-L3

TEACHER             1   0
                    2   0
                    3   0
                    4   0
                    5   0

TEACHER*METHOD 1 a      0
               1 b      0
               2 a      0.5*L2
               2 c      -0.5*L2
               3 a      0.5*L2
               3 b      L3
               3 c      -0.5*L2-L3
               4 b      0
               5 a      0
               5 b      0
```

In this case, we would be testing whether the following two contrasts are 0:

$$L_2 = \frac{\mu_{12} + \mu_{13}}{2} - \frac{\mu_{32} + \mu_{33}}{2},\ L_3 = \mu_{23} - \mu_{33}$$

This approach may seem too conservative when there are many cells missing but you at least have a good handle on the contrasts tested; the same could not be said of the Type III contrasts.

Class Exercise. Using the SAS output below, find the four contrasts being tested in the Type IV analysis for Teacher. Be sure to write them as functions of $\{\mu_{ij}\}$. Comment.

```
Type IV Estimable Functions for:  TEACHER

Effect                  Coefficients

INTERCEPT               0

METHOD            a     0
                  b     0
                  c     0

TEACHER           1     L5
                  2     L6
                  3     L7
                  4     L8
                  5     -L5-L6-L7-L8

TEACHER*METHOD  1 a     0.5*L5
                2 a     L6
                3 a     0.5*L7
                5 a     -0.5*L5-L6-0.5*L7
                1 b     0.5*L5
                3 b     0.5*L7
                4 b     L8
                5 b     -0.5*L5-0.5*L7-L8
                2 c     0
                3 c     0
```