Final Exam

1. Consider Example 10.15 from the text.
   (a) Write the Rao-Blackwell estimates (posterior means) for the conditional means of $\lambda_i$ and $\beta_i$.
   (b) For Herd 15, repeat your simulation for 10.10(b-c) from Homework 5. Compute both Rao-Blackwell estimators and the usual empirical estimators of the posterior means for both $\lambda_{15}$ and $\beta_{15}$. How effective does Rao-Blackwellization seem to be in reducing the estimated standard errors of the posterior means?
   (c) From the parameters at each iteration, compute a Rao-Blackwellized density estimate of $\pi(\lambda_{15}|x,\alpha,\beta_i)$; compare this estimate to the usual empirical estimate from a Gibbs sampler.

2. Consider the Ising model we studied in class. I have included a script that creates an image and a function that adds noise on the website.
   (a) Run the code for `znoise` to generate the initial figure. Plot the figure using `levelplot`. Now add 20% noise, and then 10% noise. Plot and comment. In the following, we will use the figure with 10% noise added.
   (b) `ising.mh` currently generates a random grid as the input matrix to the Metropolis-Hastings algorithm for the Ising model. Modify `ising.mh` so that it accepts an input grid (this requires only a minor change in the code). You should use the figure with 10% added noise as the input matrix in the following.
   (c) Run a single iteration of `ising.mh` using the default value for $\beta$; are the plotted results encouraging?
   (d) Continuing with a single iterate, experiment with different values for $\beta$ and comment on the output. Which values of $\beta$ work best? Do any of them work “well”? Which features of the figure are reliably captured by the Ising “filter”? Which are lost? Suggest modifications to improve the performance of the Ising filter.

3. Suppose we want to compute the .95 quantile ($x_{.95}$) of a Gumbel distribution:
   \[P(X \leq x) = F(x) = e^{-e^{-x/\lambda}}, \quad -\infty < x < \infty.\]
   (a) Find $F^{-1}(u)$ and write code for a random number generator using the Probability Integral Transform.
   (b) For $\lambda = 1$, generate and save random samples of size 10, 20 and 100. What are the empirical estimates of $x_{.95}$ for these samples? Compare these estimates to $x_{.95}$.
   (c) Using `boot`, generate B=200 bootstrap samples for each of your three samples and compute estimates of the bootstrap estimator, bias and standard error. Comment on your results.
   (d) Construct 95% confidence intervals for $x_{.95}$ as well, using B=1000. Compare results for basic, normal, percentile and $BC_\alpha$. 

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