Homework 2

Do 2 out of 3 methods problems, and both computer exercises.

1. Explain how you would use the Probability Integral Transform to generate random variables from the logistic distribution:

\[
f(x) = \frac{\exp\{x\}}{(1 + \exp\{x\})^2}, \quad -\infty < x < \infty.
\]

2. Robert and Casella (2004) Problem 2.31: For the Accept-Reject algorithm of the \( \Gamma(n, 1) \) distribution, based on the \( \text{Exp}(\lambda) \) distribution, determine the optimal value of \( \lambda \).

3. Kennedy and Gentle (1980) studied the following algorithm to generate Poisson deviates using the well-known relationship between exponential waiting times and Poisson counts:

1. Set \( L = \exp^{-\lambda}, x = 0, p = 1 \).
2. Generate \( u \) from \( \text{Unif}(0, 1) \) and set \( p = pu \).
3. If \( p \geq L \), set \( x = x + 1 \) and repeat step 2. Otherwise, output \( x \).

Explain why this algorithm generates Poisson deviates. Hint: If \( N \sim \text{Pois}(\mu) \), and \( X_1, \ldots \) are iid \( \text{Exp}(1/\mu) \), show that \( N = n \iff \sum_1^n X_i \leq 1 \) and \( \sum_1^{n+1} X_i > 1 \).

**Computer Project.** Write a program to generate random deviates from a normal distribution using the rejection method. Use the logistic function from Problem 1 with \( c = 4/\sqrt{2\pi} \). The program should allow the user to specify the number of deviates. The output list should contain the vector of normal deviates and the proportion of acceptances. Write a second program to generate Poisson deviates using Kennedy and Gentle’s approach in the above problem. The program should allow the user to specify the number of deviates and the Poisson mean. The output vector should contain the Poisson deviates.