Midterm Exam

1. Mid P-value

   (a) Show that the lower confidence bound π_L for the binomial parameter π using the mid-P-value method must solve the following equation (I've left the 1/2 term in the equation to help with the derivation):

$$\alpha/2 = \frac{1}{2}F\left(\frac{\pi_L(n-y)}{(1-\pi_L)(y+1)}; 2(y+1), 2(n-y)\right) + \frac{1}{2}F\left(\frac{\pi_L(n-y+1)}{(1-\pi_L)y}; 2y, 2(n-y+1)\right)$$

   (b) Find the solution for this equation when $y = 3$, $n = 25$, and $\alpha = .05$.

2. Clopper-Pearson confidence intervals.

   (a) Modify code for the Clopper-Pearson confidence interval so that it generates confidence intervals for input values of y and n (rather than y and $n = 25$ as we used in class and HW 1).

   (b) Compute 95% confidence intervals for $y = 1$ as n varies from 2 to 100. Plot the lower and upper confidence bounds as a function of n.

   (c) Plot the actual coverage probability for 95% Clopper-Pearson confidence intervals for $\pi = .05$ as n varies from 2 to 100. Comment.

   (d) Write code to compute actual coverage probabilities for 95% Clopper-Pearson confidence intervals as n varies from 5 to 100 in increments of 5, and π varies from .01 to .5 in increments of .01. Display results using contour plots, or wireframe plots or the 3-dimensional method of your choice (the `lattice` package has a variety of 3-dimensional plotting functions). It may be useful to subtract actual coverage proportions from .95 and instead plot these deviations from the target value. Comment.

3. Consider the following 2X2 table (with row and column marginals included):

10	7	17
11	2	13
21	9	30

   (a) For Fisher's Exact test of independence, what is the range of n_{11}? Using SAS, what is the p-value for Fisher's Exact test for $H_A:\ \theta < 1$?

   (b) Compute Pearson's chi-squared test statistic for independence. Find the exact p-value for this test.

4. Use the same table as in Problem 4 to answer the following questions.

   (a) Consider a test of $H_o:\ \theta = 2$ vs. $H_A:\ \theta < 2$ using the method in 3.6.1 Write R code to find a p-value for this test.

   (b) Using two one-sided tests, each with $\alpha = .025$, write R code to find a 95% confidence interval for θ.