STAT 515 hw 2

Counting, conditional probability, Bayes' rule, independence

1. Download the golden ratio data from the course website. Then read it into R with this command (you will have to change the path to the file depending on where you save it):

```
data <- read.csv(file = "pathtothedirectory/golden_ratio_sp_2024.csv")
```

   Let $X = B/A$; that is, divide each B measurement by the corresponding A measurement to create a new variable called X. Then

   (a) Make a boxplot of the X values with the `boxplot()` function.

   (b) Compute the mean $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ and the sample variance $S_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2$ of the X values. You can use the functions `mean()` and `var()`.

   (c) Make a histogram of the X values using the `hist()` function.

2. An iteration of the board game Candyland has the following cards: 6 each of red, purple, yellow, blue, orange, and green cards, and then 4 each of double red, double purple, double yellow, double blue, double orange, and double green cards. In addition, there is one lollipop, one peppermint, one peanut, and one ice cream card.

   (a) If you draw 5 cards, one at a time, placing each card back into the deck and shuffling before the next draw, how many sequences of draws are possible?

   (b) If you draw 5 cards, one at a time, without placing each card back before drawing the next, what is the probability that one of your 5 cards will be the ice cream cone?

3. Three guests to a tea party sit down in three chairs. Then the host re-arranges them according to a pre-made seating chart

   (a) What is the probability that all of the guests may stay in their chairs?

   (b) What is the probability that at least one guest may stay in her chair? *Hint: Just write down all the sample points. Don't try to use counting rules.*

4. A train to the western frontier will consist of 4 passenger cars, 3 cattle cars, and 2 luggage cars, which are to be put in order at random.

   (a) Bandits plan to mount the train and enter the two rearmost cars. Find the probability that they enter

      i. two passenger cars.

      ii. a luggage car and a passenger car.

      iii. at least one cattle car.

   (b) A lady's shawl flies out a window of the foremost passenger car, reenters a window in the rearmost passenger car, and is seized by a gentleman who gallantly vows to return it while the train is in motion. With what probability can he make his way from the rearmost passenger car to the foremost, passing only through passenger cars?

   (c) There are 14 head of cattle to be transported in the three cattle cars.

i. In how many ways can 5, 5, and 4 head of cattle, respectively, be put into the three cattle cars?

ii. The gallant gentleman owns 3 of the 14 head of cattle. If 5, 5, and 4 of the 14 head of cattle are put into the three cattle cars at random, with what probability will the gallant gentleman's cattle all be placed in the same cattle car?

(d) Mr. and Mrs. Wilkins and their two daughters and three sons are boarding one of the passenger cars on the next stop.

i. In how many different orders can the members of the Wilkins family enter the car?

ii. In how many of these orders does Mrs. Wilkins precede Mr. Wilkins?

iii. In how many of these orders does Mrs. Wilkins and her two daughters precede Mr. Wilkins his three sons?

(e) At each of the 10 stops along Lil' Jonnie's journey on the train, he will decide whether to quickly hop out and scrawl his initials somewhere around the station or to remain on board. At how many unique sets of stations can Lil' Jonnie scrawl his initials?

5. Suppose there are 5 bowling balls which are identical except that one is magical and delivers, no matter what, a strike with probability 3/4. Suppose you get a strike 1 out of 4 times on average when using non-magical bowling balls. You select one of the 5 balls at random and send it down the lane...

   (a) Give the probability that you get a strike.

   (b) Given that you got a strike, what is the probability you chose the magic bowling ball?

   (c) Suppose you choose a ball and with the same ball you get two strikes in a row. What is the probability that you chose the magic ball?

6. Consider a bag of marbles, 19 of which are green, 25 of which are blue, and 6 of which are red. Moreover, suppose 9 of the green marbles are opaque, 5 of the blue marbles are opaque, and 3 of the red marbles are opaque, and the rest of the marbles are transparent. Suppose you draw one marble from the bag and let G, B, and R be the events that the marble is green, blue, and red, respectively, and let O be the event that it is opaque.

   (a) Find $P(R|O)$.

   (b) Find $P(R|O^c)$.

   (c) Find $P(R)$.

   (d) Check whether R and O are independent.

   (e) Find $P(B^c|O)$.

   (f) Find $P(O|G)$.

   (g) Find $P(B \cup G|O)$.

7. In all the state of the union (SOTU) addresses since that of President George Washington in the year 1790 to that of President Barack Obama in the year 2016, a total of 69,158 sentences were spoken. In all SOTU addresses following that of President Woodrow Wilson in 1920, a total of 32,752 sentences were spoken; among these, 15,934 had a length or 20 words or more. Among the

sentences spoken prior to this, 27,639 had a length of 20 words or more.
See https://doi.org/10.46430/phen0061 *if you want to play with this kind of data.*

(a) If you select a sentence at random from the SOTU addresses given after 1920, with what probability does it have a length of at least 20 words?

(b) If you select a sentence at random from the SOTU addresses given before or during 1920, with what probability does it have a length of at least 20 words?

(c) If you select a sentence at random from all the SOTU addresses from 1790 to 2016, with what probability does it have a length of at least 20 words?

(d) If you select a sentence at random from all the SOTU addresses from 1790 to 2016, what is the probability that it belongs to a SOTU address given after 1920, given that it has a length of at least 20 words?

8. Suppose you order a new pair of swimming goggles and that they will be manufactured with a defect making them leaky with probability 1/40.

(a) What is the probability of receiving a defective pair, and then again receiving a defective pair when you re-order the goggles?

(b) What assumption did you make in order to compute your answer to part (a)?

(c) What is the probability that you receive a defective pair and then a functioning pair when you re-order the goggles?

(d) Give the probability in terms of K that you receive K defective pairs of goggles, where $K \geq 1$.