STAT 515 hw 8

Hypothesis testing for the mean and proportion by comparing test statistics to critical values

1. Each of fifteen students measured a cup of flour by fluffing the flour and then scooping it into a measuring cup with a spoon until it was slightly mounded above the brim, and then scraping away the excess. The student then measured the weight of the cup of flour in grams, resulting in the following measurements:

   `128 151 125 127 128 117 130 129 127 154 162 130 154 131 140.`

   Assume that the measurements follow a Normal distribution.

   (a) Write down the hypotheses of interest for the following chefs:

      i. Chef Boyardee wishes to know whether the mean weight of cups of flour measured in this way exceeds 120 grams.

      ii. Chef Bertolli wishes to know whether the mean weight of cups of flour measured in this way is less than 120 grams.

      iii. Chef Ragu wishes to know whether the mean weight of cups of flour measured in this way differs from 120 grams.

   (b) Suppose the true mean weight of cups of flour measured in this way is 115 grams, and that each of the chefs in part (a) conducts a study to answer his question.

      i. For which of the chefs in part (a) is a Type I error possible?

      ii. For which of the chefs in part (a) is a Type II error possible?

   (c) Assume that the standard deviation of the weights of cups of flour measured in this way is $\sigma = 12$.

      i. Give a 95% confidence interval for the mean weight of cups of flour measured in this way.

      ii. Give the value of the test statistic for testing the hypotheses of the chefs in part (a).

      iii. State, for each chef in part (a), whether he rejects his null hypotheses at the $\alpha = 0.05$ significance level based on the class data. Explain your answer.

   (d) Assume that the standard deviation of the weights of cups of flour measured in this way is unknown and use the sample standard deviation.

      i. Give a 95% confidence interval for the mean weight of cups of flour measured in this way.

      ii. Give the value of the test statistic for testing the hypotheses of the chefs in part (a).

      iii. State, for each chef in part (a), whether he rejects his null hypotheses at the $\alpha = 0.05$ significance level based on the class data. Explain your answer.

2. Rosenzweig et al. (1972) subjected pairs of rats to live in "impoverished" versus "enriched" surroundings, where each pair of rats was taken from a single litter to ensure genetic similarity. In the enriched environment, rats lived among other rats in large cages and were furnished with new playthings everyday, whereas in the impoverished environment, each rat lived alone in an unfurnished cage. At the end of an experimental period "the rats were sacrificed and their brains removed" and their cortexes weighed, resulting in the following weights in milligrams (partial data are given as presented on pg. 453 of Freedman et al. (1991)):

Pair	Enriched	Impoverished	Pair	Enriched	Impoverished
1	689	657	7	664	600
2	656	623	8	647	640
3	668	652	9	694	605
4	660	654	10	633	635
5	679	658	11	653	642
6	663	646			

(a) Write down the hypotheses in which the following scientists are interested:

  i. Rosenzweig wants to know if the enriched environment leads to heavier cortexes.

  ii. Bennett wants to know if the enriched environment results in lighter cortexes.

  iii. Diamond wants to know if being in an enriched versus an impoverished environment has any effect on the weights of rat cortexes.

(b) Suppose the mean difference (enriched minus impoverished) is 5 milligrams.

  i. Which scientists could make a Type I error?

  ii. Which scientists could make a Type II error?

(c) Calculate the sample mean and standard deviation of the differences between the cortex weights of the rats in the enriched versus impoverished environments (enriched minus impoverished).

(d) Make a QQ plot of the differences between the cortex weights of the rats in the enriched versus impoverished environments (enriched minus impoverished). Comment on whether the differences appear to be Normally distributed.

(e) Assuming that the differences in cortex weight follow a Normal distribution, give the value of the test statistic for testing the hypotheses of the scientist in part (a).

(f) Assuming that the differences in cortex weight follow a Normal distribution, state, for each scientist in part (a), whether he or she rejects the null hypothesis at the $\alpha = 0.05$ level.

(g) Assuming that the differences in cortex weight follow a Normal distribution, build a 95% confidence interval for the mean difference in cortex weight between rats in an enriched versus an impoverished environment.

3. Sea lampreys are an invasive species in the Great Lakes—primarily in Lake Michigan—whose threat to the population of fish there began causing alarm in the 1930s. Sea lampreys attach themselves to fish and feed on them. In 1947, Vernon C. Applegate caught 3,700 white or redhorse suckers (kinds of fish) in a weir in a tributary to Lake Michigan, among which 259 had scars inflicted by sea lampreys (Applegate, 1950).

(a) Suppose the Department of Conservation will take measures to control the sea lamprey population in a tributary if the proportion of scarred white or redhorse suckers in a tributary is determined to exceed 0.06.

  i. State the null and alternate hypotheses of interest to the Department of Conservation.

  ii. Given the data of Vernon C. Applegate, would the Department of Conservation take measures to control the sea lamprey population if using the significance level $\alpha = 0.05$?

  iii. If using the significance level $\alpha = 0.005$?

(b) Suppose measures to control the sea lamprey population are already underway in a tributary and that the Department of Conservation will cease these measures if the proportion of scarred white or redhorse suckers in the tributary is determined to be less than 0.02.

   i. State the null and alternate hypotheses of interest to the Department of Conservation.

   ii. Given the data of Vernon C. Applegate, would the Department of Conservation cease its measures of controlling the sea lamprey population if using the significance level 0.05?

   iii. If using the significance level $\alpha = 0.005$?

(c) Given the data of Vernon C. Applegate, construct a 95% confidence interval for the proportion of white or redhorse suckers in the tributary on which sea lampreys have fed.

4. Last summer you earned \$120 every Saturday at your job and you are planning to take the same job again this summer. Your friend has a job with wages coming mostly from tips, and brags about making \$222 on one Saturday. She can get you a job where she works this summer if you want. In order to base your decision on rigorous statistical inference, you collect from her the wages she earned on several Saturdays last summer, getting the amounts

115.15	117.34	222.09	204.05	80.04	78.46
162.54	172.99	181.03	135.68	192.70	122.28.

Let μ denote the expected earnings on a Saturday at the job your friend can get for you.

(a) You decide that if you can expect to earn over \$20 more on a Saturday working where your friend works, then you will ask her to get you a job there. What are the relevant hypotheses?

(b) Suppose you really like your old job and you only want to switch if you are quite quite sure that you will make over \$20 more on average. Then, referring to the hypotheses in part (a), you should perform your test

   A. with a large α to ensure a small probability of Type I error.

   B. with a small α to ensure a small probability of Type I error.

   C. with a large α to ensure a small probability of Type II error.

   D. with a small α to to ensure a small probability of Type II error.

(c) Assuming your friend's wages are Normally distributed, perform the test of your hypotheses in part (a) at the $\alpha = 0.05$ significance level.

(d) You discover that your old supervisor quit and you don't really want to work under the new supervisor. You decide that you will ask your friend to get you a job where she works unless you can conclude that you will make more on Saturdays, on average, at your current job. What are the relevant hypotheses now (still letting μ represent the amount of money you could expect to make at your friends job on a Saturday)?

(e) Interpret what it means if you commit a Type I error when testing the hypotheses in part (d).

Optional (do not turn in) problems for additional study from McClave, J.T. and Sincich T. (2017) *Statistics*, 13th Edition: 8.33, 8.35, 8.38, 8.40, 8.60, 8.61, 8.62, 8.74, 8.76, 8.78, 8.80, 8.88.

References

Applegate, V. C. (1950). *Natural history of the sea lamprey, Petromyzon marinus, in Michigan.* PhD thesis, University of Michigan.

Freedman, D., Pisani, R. L., Purves, R., and Adhikari, A. (1991). *Statistics, Second Edition.* W.W. Norton & Company, Inc.

Rosenzweig, M. R., Bennett, E. L., and Diamond, M. C. (1972). Brain changes in response to experience. *Scientific American*, 226(2):22–29.